FILM CONDENSATION ON HORIZONTAL SURFACES

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CERTIFICATE

This is to certify that the present work on 'Film Condensation on Horizontal Surfaces', has been carried out by Mr. Vishwanath Prasad, under my supervision and has not been submitted elsewhere for a degree.

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In fond memory of
my mother

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TABLE OF CONTENTS

LIST OF FIGURES

Page

NOMENCLATURE

A	Area of the condensing surface
b	Thickness of the plate
B	$\sigma/(\rho - \rho_v)g$
Bi	Biot Number $= hb/k_p$
Bo	Bond Number $= \bar{\rho}\, gL^2/\sigma$
C_p	Specific heat
g	Acceleration due to gravity
h	Heat transfer coefficient
h_{fg}	Latent heat of vapour
h'_{fg}	$h_{fg} + \frac{3}{8} C_{p_1} \Delta t$
k	Thermal conductivity
K	$\dfrac{3\, \nu k_1 \Delta t}{\bar{\rho}\, g\, h'_{fg}}$
L	Length of the condensing surface
$(\dot{m})_{in}$	Mass rate of condensation
$(\dot{m})_{out}$	Mass rate of condensate flow
Nu_x	Local Nusselt Number $= h_x\, x/k$
Nu_m	Mean Nusselt number
p	Pressure
P	Perimeter of the condensing surface allowing the flow of condensate
Pr	Prandtl Number $= C_p\, \mu/k$

q	Heat flux
r	Fraction of the condensate, condensed at any time, flowing out across the edges of the condensing surface at that time
R	$\dfrac{3\rho\nu\left(1+\frac{1}{8}\,\frac{C_{p_l}\,\Delta t}{h'_{fg}}\right)L^2}{\bar{\rho}\,g}$
R_x	Radius of curvature
Sh	Sherwood Number = $\dfrac{\bar{\rho}\,g\,h'_{fg}\,L^3}{\nu\,k_l\,\Delta t}$
t	Temperature
Δt	$t_s - t_i$
u	Velocity in x-direction
x	Direction of heat flow as shown in Figs. 1-5.
x, y	Co-ordinate system as shown in Fig. 22.
$\bar{x}, \bar{y}$	Co-ordinate system as shown in Fig. 23.
X	Dimensionless location (a) $X = x/b$, for transient film condensation (Chapter II) (b) $X = x/L$, for laminar film condensation flow (Chapter III)
ρ	Density of the condensate
ρ_p	Density of the plate material
ρ_v	Density of the condensing vapour
$\bar{\rho}$	$\rho - \rho_v$

α	Thermal diffusivity of the plate material
β	Condensation parameter $= \sqrt{\dfrac{\rho\, h_{fg}\, \alpha\, k_1}{k_p^2\, \theta_s}}$
γ	Gravity flow parameter for transient film condensation $= \frac{2}{3} \sqrt{2g}\, \frac{P}{A}\, \frac{b^2}{\alpha} \left(\frac{b^2}{\alpha}\, \frac{k_1\, \theta_s}{\rho\, h_{fg}}\right)^{1/4}$
δ	Condensate film thickness
λ	Dimensionless film thickness $= \dfrac{\delta}{\left(\frac{b^2}{\alpha}\, \frac{k_1\, \theta_s}{\rho\, h_{fg}}\right)^{1/2}}$ (Chap.II)
η	Dimensionless film thickness $= \delta/\delta_{max}$ (Chapter III)
Δ	Penetration depth
$\bar{\Delta}$	Dimensionless penetration depth $= \Delta/b$
τ	Time
ξ	Dimensionless time $= \dfrac{\tau}{b^2/\alpha}$
θ	Excess temperature $= t - t_i$
$\emptyset$	Dimensionless temperature $= \dfrac{t - t_i}{t_s - t_i}$
μ	Viscosity of the condensate
ν	Kinematic viscosity of the condensate
σ	Surface tension

Superscripts and Subscripts

i	Initial condition
l	Condensate (liquid) region
max	Maximum value
min	Minimum value
o	Condition at the condensing surface
p	Plate region
s	Saturation condition of the vapour
v	Vapour region
x	At any location x
σ	Surface tension case

ABSTRACT

Transient film condensation on horizontal surfaces and laminar film condensation flow on a horizontal isothermal surface are considered. Transient film condensation on horizontal surfaces is analyzed for various boundary conditions of the plate. These are: (a) isothermal surface, (b) plate with insulated bottom surface and a negligible temperature variation across its thickness, (c) semi-infinite approximations, (d) plate of finite thickness and bottom surface at a fixed temperature, and (e) plate of finite thickness and bottom surface insulated. Each case is considered for two inviscid flow conditions, one, a specified fraction of the rate of condensation, occurring at any time, flowing out across the edges of the condensing surface at that instant, and, second, the outflow dominated by gravity. The appropriate conservation equations are simplified by using several approximations valid for many physical circumstances of interest, and the resulting ordinary non-linear differential equations, except for the first type of flow in case (b), where exact solution is obtained, are integrated numerically. The theoretical results are shown to depend on two dimensionless parameters, one for the condensation, except for the case of isothermal surface, and, the other for the flow. The situation of no-flow arises when the flow parameter is zero.

Laminar film condensation flow on a horizontal isothermal surface is analyzed in three stages: (a) transient film condensation without flow, (b) transient laminar flow, and (c) steady, laminar flow considering the surface tension effects, fourth order non-linear differential equations are obtained for transient and steady flows. Negligible surface tension, or large values of Bond Number reduce them to second order. The condensate film profiles for steady laminar flows are found to be strong functions of edge conditions at the end and the length of the condensing surface, and are also observed to depend on the vapour properties and the difference between the temperature of the vapour and that of the condensing surface. Results obtained show that for the condensing surface, terminated by a round fall, the surface tension effects are negligible, even for very small values of the Bond number. The analysis and results for transient, laminar flow show that the transient effects are large for highly viscous condensing fluids, or for low gravity field. A very simple expression for the local Nusselt number is also obtained.

CHAPTER I

INTRODUCTION

The importance of the condensation heat transfer process is due to the large number of applications it finds in industry. For example, the condensation process is of considerable importance in the design and study of the condensers of thermal power stations, thermal processing systems, nuclear power generation systems, refrigeration and air-conditioning systems and in many manufacturing processes, which employ the condensation of a hot vapour for the thermal treatment process. Condensation soldering, curing, heat treatment and degreasing processes are few examples of the application of condensation in manufacturing industry.

A vapour in contact with a surface at a temperature lower than the saturation vapour temperature, corresponding to the given vapour pressure, will condense on the surface. The condensate thus formed will be in the subcooled state due to contact with the cooled surface, and more vapour will condense on the exposed surface and on the previously formed condensate. Depending on the behaviour of the condensate, with respect to the cooled surface, the condensation process has been divided into two distinct condensation mechanisms. If the condensate tends to wet the surface, thereby forming a liquid

film, the process is termed film condensation. If the condensate does not wet the surface, but, instead, collects in growing droplets on the cooled surface, the process is termed dropwise condensation. Both types of condensation are common, and, in some circumstances, may be simultaneously encountered on a single surface. Due to the direct contact between the vapour and the surface and the extended - surface effect of the droplets the heat transfer rates are much larger in the case of dropwise condensation, as compared to those in film condensation, where the film introduces a thermal resistance.

If the condensate forms a film over the cooling surface, the process is quite different. The heat transferred to the cooled surface, from the vapour, occurs by the convective heat transfer across the liquid film, the higher temperature being at the interface, which is in contact with the vapour. This convection process may greatly reduce the overall heat transfer rate, depending upon the thickness of the film, the fluid properties and turbulence in the flow. Therefore, the heat transfer rates for film condensation are considerably lower than those for dropwise condensation, and in many applications, efforts are made to generate a dropwise condensation by vibrating the cooled surface.

Most of the condensation processes, generally encountered, give rise to film condensation or are mixed in character. Since a developed, steady, laminar film will also give the lowest

steady heat flux, one would have to contend with in any design; a study of the film condensation process takes on a special importance. Almost any departure from the simple film condensation process will give rise to a higher heat flux, at a given temperature difference between the surface and the vapour, Δt. In transient processes the heat transfer coefficient varies with time and may give a minimum value as observed in transient natural convection flows.

The physical nature of the steady laminar film condensation process is well understood, due to the many previous and recent investigations. The steady laminar process was first considered in 1916 by Nusselt [1] and his predictions were found to be in good agreement with experimental results for condensing fluids of Prandtl Number, Pr_1, around unity. This simple analysis ignored acceleration, or inertia, effects, assumed a linear temperature distribution in the film and neglected energy effects with liquid subcooling and interfacial shear between the liquid film and the vapour. Subsequent analysis by Bromley [2], and by Rohsenow [3] dispensed with some of these approximations. The results are identical with those of the Nusselt theory, except that h_{fg} in those relations is replaced by $h_{fg} + \frac{3}{8} C_{p_1} \Delta t$; where C_{p_1} and h_{fg} are the specific heat of the condensate liquid and the latent heat of the vapour, respectively. The correction $\frac{3}{8} C_{p_1} \Delta t$ is not negligible for some condensation processes and gives results

which deviate from those obtained by Nusselt [1] for large values of $C_{p_1} \Delta t/ h_{fg}$. In practice, this value is usually less than 0.1 [4].

A boundary layer similarity analysis, neglecting the interfacial shear, was carried out by Sparrow and Gregg [4,5], without making any of the other three assumptions of Nusselt, listed above. For Prandtl Number unity and above and $C_{p_1} \Delta t/h_{fg} \leq 1$, the results obtained by Sparrow and Gregg [4,5] are quite close to those obtained by omitting the inertia terms But inertia terms play a significant role in the condensation heat transfer process for Prandtl Number values much less than unity, generally for $Pr_1 < 0.05$. The results of Nusselt [1], Rohsenow [3] and Sparrow and Gregg [4] have been compared graphically by Gebhart [6].

Other film condensation analyses, considering the shear at the liquid - vapour interface, were presented by Koh et al.[7], Chen [8] and Poots and Miles [9]. The results show that the effect is very small for Prandtl Numbers of 10 or greater. The effect increases with the subcooling parameter $C_{p_1} \Delta t/h_{fg}$ and, for very low Pr_1, the reduction in heat transfer may be as large as 30 percent, for $C_{p_1} \Delta t/h_{fg}$ as large as 0.1. Rohsenow and Hartnett [23a] have presented the comparison of results of Chen [8] and Sparrow and Gregg [4,5]. Yang [10] used a series technique for film condensation with variable surface temperature and obtained solutions for a

range of Prandtl Number. Sparrow and Lin [11] analyzed the effects of noncondensable gases present in the condensable vapour and reported that the presence of a very small amount of noncondensable gas reduces the heat transfer coefficient significantly. Minkowycz and Sparrow [12] considered the effect of superheated vapour and have reported that the rate of condensation is slightly increased by the superheat in vapour. Sukhatme and Rohsenow [13] and Mills and Seban [14] have shown that the interfacial resistance at the vapour liquid interface is not important for many fluids and practical conditions. Steady, laminar film condensation, in the absence of the gravitational field, has also been investigated by Koh [15], Sparrow et al. [16], Minkowycz and Sparrow [17] and Gebhart [19].

There is apparently much less in literature on the transient flow and on laminar film condensation arising on horizontal surfaces. These situations are typically much more complicated than those described above. However, transient and steady film condensation on finite horizontal surfaces are important in technology and in nature, as, for instance, during the start-up of systems and in a number of other processes of interest where the horizontal surface is immersed in a condensing vapour region at a given time.

Sparrow and Siegel [18] were the first to analyze the transient film condensation flow on vertical isothermal

surfaces. Their analysis ignores inertia effects and assumes a linear temperature profile in the condensate film. Nimmo and Leppert [20, 21] have analyzed the laminar film condensation flow over horizontal isothermal surfaces neglecting the inertia, momentum and surface tension effects, and assuming a linear temperature distribution in the condensate film and zero shear at the vapour - liquid interface.

Gebhart [19] and Rohsennow and Hartnett [23b] have considered the transient film condensation over isothermal horizontal surfaces with no run-off, i.e., no flow of condensate over the edges, and have found that the condensate film thickness δ is given by, $\delta = K_1 \tau^{1/2}$, where τ is the time and K_1 a constant, which depends on the vapour and condensate liquid properties and on Δt. The latter investigators have also analyzed the transient condensation problem, with zero gravity. All these analyses are for a uniform wall temperature, assuming a saturated vapour and neglecting the resistance at the liquid-vapour interface, the momentum and convection effects and the variation of fluid properties with temperature.

Pfahl [22] has shown that some of the conduction solutions, available in Carslaw and Jaeger [26], for problems with a change of phase, can be employed in the modelling of liquid - vapour condensation problems, provided that there is no motion in the liquid phase. In particular, he has

considered two exact solutions, for transient film condensation on horizontal surfaces; one, on an isothermal surface, and the other, on a semi-infinite container bottom, initially, at temperature $t_i < t_s$. In both the cases, the condensate film thickness δ increases with time and is given by, $\delta = K_2 \tau^{1/2}$, where K_2 is a constant and depends on the vapour, condensate and surface properties and on Δt. Recently, Mollendorf and Chu [24] have analyzed the transient film condensation over a horizontal body, initially at t_i ($< t_s$), for an insulated underside and a low value of the Biot Number, i.e., negligible temperature variation across the thickness of the body. In their analysis, they have considered the gravity - dominated flow of the condensate. Their analysis is also subject to the usual approximations, mentioned above, and neglects the surface tension effects.

The present work is directed at an analysis of film condensation on horizontal surfaces. The work is divided into two main parts. The first concerns the transient film condensation on a horizontal body of uniform thickness, with run-off, and the second concerns the laminar film condensation flow on a horizontal isothermal surface.

For the transient film condensation on horizontal plates, the analysis has been carried out for five different cases. These are related to flow and heat transfer for the following conditions, assuming condensation to occur on the

upper surface:

(i) An isothermal surface

(ii) A plate with insulated bottom surface and a negligible temperature variation across its thickness.

(iii) Semi-infinite Approximation: Solution for short times and thick plates.

(iv) A plate with a finite thickness and bottom surface maintained at a constant temperature.

(v) A plate with a finite thickness and an insulated lower surface.

Furthermore, each case has been analyzed for two types of flow:

I. A specified fraction of the rate of condensation, occurring at any time, assumed to flow out across the edges of the condensing surface.

II. Gravity dominated flow, in which the outflow velocity is determined by the gravitational force resulting from the height of the condensed fluid.

The results obtained show that the heat transfer coefficient is a strong function of time and is infinite at time $\tau = 0$, and decreases exponentially to zero for large τ. A dimensionless condensation parameter β, depending on the fluid and the plate properties, $(t_s - t_i)$ and h_{fg}, has been found, which controls film condensation on horizontal

plates, for all boundary conditions considered except for the isothermal surface. For no flow and for a specified fraction of the condensate flowing out, the condensate film thickness increases with time except for the cases in which lower surface of the plate is insulated. For the plate with insulated bottom surface, the film thickness achieves a constant value after a large time, when the temperature of the plate has reached a value close to the saturation temperature of the vapour. In the case of the semi-infinite solution, the temperature of the upper surface of the plate picks up a constant value, depending on the fluid and the plate properties, as soon as it is exposed to the condensing vapour. In all other cases the temperature in the plate is a strong function of time except for the isothermal surface. For no flow condition, the results obtained for an isothermal surface and for thick plates are in good agreement with the exact solutions presented by Pfahl [22].

For gravity dominated flow, another dimensionless flow parameter γ has been obtained, which, along with β, describes the condensation process over horizontal plates. In the present case, the condensate film thickness first increases with time and then becomes constant, the rate of condensation being equal to the rate of flow, except for the cases of insulated lower surface, where the film thickness becomes almost zero, the surface temperature being very close to t_s.

In the second part of the present work, an analysis of the laminar film condensation flow on a horizontal isothermal surface has been carried out. The following three stages have been considered.

1. Transient film condensation without flow.
2. Transient, laminar film condensation flow.
3. Steady, laminar film condensation flow.

Non-linear differential equations of fourth order have been obtained for transient and steady state laminar film condensation flow with appreciable surface tension effects. For large values of the Bond Number, Bo, or negligible surface tension, they reduce to second order systems. The differential equation obtained for steady laminar flow with large Bo is the same as the equation obtained by Nimmo and Leppert [20, 21].

For steady, laminar flow, the results obtained show that the condensate film profile is a strong function of edge condition, i.e., η_{min}, the dimensionless film thickness at the end, and the length of the condensing surface, L. While the rate of condensation, and, hence, the flow rate increases as L is increased, they decrease with increased η_{min}. The parameter K, which is a function of the properties of the condensing vapour and Δt, also affects the film profile. The condensate film is found to be thicker for a higher value of

K, and, hence, reduces the flow rate. The local Nusselt Number Nu_x is equal to $x/\delta(x)$ where x is the location and $\delta(x)$ is the film thickness at x [Fig.22].

For transient laminar flow, a parameter R has been obtained which is large for highly viscous condensing fluids, or for low gravity, and is found to increase with the length of the condensing surface, L, and $C_{p_l} \Delta t / h'_{fg}$. The rate of increase in film thickness decreases with time and, hence, the local heat transfer coefficient h_x is also reduced. The surface tension effects are found to be negligible for the condensing surface terminated by a round fall.

All the computations are performed on IBM 7044 computer at IIT Kanpur. For the numerical solution of ordinary differential equations, for transient film condensation, combined with conduction, on a horizontal body (Chapter II), 4th order Runge-Kutta method is used and for the ordinary and partial differential equations, developed for the laminar film condensation flow, the finite - difference technique is employed.

CHAPTER II

TRANSIENT FILM CONDENSATION, COMBINED WITH CONDUCTION, ON A HORIZONTAL BODY

2.1 Analysis

If the upper surface of a horizontal body, initially at temperature t_i, is subjected to a saturated vapour, at a uniform temperature t_s, corresponding to its pressure, condensation will take place on the surface provided $t_i < t_s$. The rate of condensation depends on the imposed boundary conditions for the body, besides other factors, such as fluid properties, latent heat of vapour, temperatures etc. This chapter considers the analysis and the results for transient film condensation on a horizontal surface with flow across its edges. The complete analysis has been divided into five parts, each part related to a particular circumstance of the imposed boundary conditions, for the horizontal body of uniform thickness. All these different cases are often of interest in many practical problems related to transient film-condensation on horizontal surfaces. These cases have already been mentioned in the previous chapter.

The analysis proceeds by applying the principle of mass continuity to the condensate film and that of conservation of energy to the horizontal plate. A similar analysis for the

steady state film condensation on a vertical isothermal surface was first carried out by Nusselt [1]. Several of the results obtained from the boundary layer analysis of Sparrow and Gregg [4], for steady, laminar film condensation on a vertical isothermal surface, are of interest to the present work. In particular, the analysis presented in this chapter is subject to the following approximations, besides any other mentioned in the course of analysis.

(a) No interface resistance at vapour-condensate interface, i.e., the vapour-liquid interface is at the vapour saturation temperature.

(b) The interfacial shear between the liquid film and the vapour is negligible.

(c) The vapour is saturated and is at a constant temperature corresponding to the pressure.

(d) The physical properties of the liquid and the vapour are constant with respect to temperature variation.

(e) No surface resistance to heat transfer at the condensing surface.

(f) Negligible inertia and surface tension effects.

(g) The temperature distribution in the condensate film is linear.

The assumptions (a) to (f) are very common in condensation analysis [1-5]. Regarding the last assumption, it

has been shown by Rohsenow [3] that, for the range of parameters encountered in most applications, a linear temperature profile serves as an excellent approximation. Also, Sparrow and Gregg [4] have shown that the film temperature distributions are essentially linear for small values of $C_{p_l} \Delta t/h_{fg}$, in steady flows. That all these steady state findings can be carried over to the transient situation can be inferred from the results from a study of the problem of transient ice-formation [30].

2.1.1 Isothermal Surface:

This is a very simple situation, of transient film condensation, in which a surface maintained at a temperature t_i is exposed to a saturated vapour at a constant temperature t_s at time $\tau = 0$ (Fig. 1). With the approximations discussed

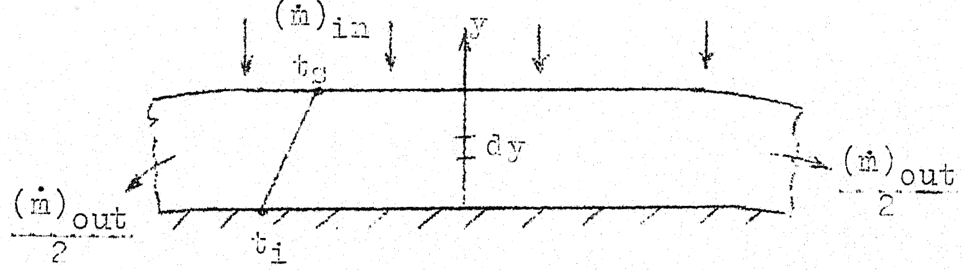

Fig. 1: Schematic showing the physical model of transient film condensation on a horizontal isothermal surface.

discussed earlier, the problem is analyzed for the two types of flow.

First, a specified fraction of the rate of condensation, occurring at any time, flowing out across the edges of the

condensing surface is considered. It is assumed that the condensate which flows out is at the saturation temperature t_s. The mass of vapour being condensed at any time is, thus, given by:

$$(\dot{m})_{in} = \rho A \frac{d\delta(\tau)}{d\tau} / (1-r) \qquad (2.1)$$

where δ is the condensate film thickness at any given time, A the area of the condensing surface, ρ the density of the condensate liquid, and r the fraction of the condensate, condensed per unit time, at a given time, flowing out across the edges of the condensing surface of the body at that time.

Assuming that the latent heat released at the vapour-liquid interface is conducted through the condensate film without any loss, we obtain,

$$\frac{\rho h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} = k_1 \left[\frac{t_s - t_i}{\delta(\tau)} \right] \qquad (2.2)$$

Integrating the above equation and using the condition that at time $\tau = 0$, $\delta = 0$, since condensation starts at $\tau = 0$, we get,

$$\delta(\tau) = \sqrt{\frac{2k_1 (t_s - t_i)}{\rho h_{fg}} (1-r) \tau} \qquad (2.3)$$

For the no flow condition, i.e., $r = 0$, the Eq. (2.3) is the same as that obtained by Gebhart [19], and by Rohsenow and Hartnett [23b].

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Now, for generalization of the analysis, we define the temperature excess θ as:

$$\theta = t - t_i \tag{2.4}$$

and non-dimensionalize δ and τ with the characteristic length δ_c and time τ_c, respectively, as:

$$\text{Dimensionless film thickness,} \quad \lambda = \frac{\delta}{\delta_c} \tag{2.5}$$

and

$$\text{Dimensionless Time,} \quad \xi = \frac{\tau}{\tau_c} \tag{2.6}$$

Here,

$$\delta_c = \sqrt{\frac{b^2}{\alpha} \frac{k_1 \theta_s}{\rho h_{fg}}} \tag{2.7}$$

$$\tau_c = \frac{b^2}{\alpha} \tag{2.8}$$

where b and α are the thickness and the thermal diffusivity of the plate.

It may be noted that for an isothermal surface, the variables b and α do not enter the analysis. However, this non-dimensionalization has certain advantages in the other cases to be analyzed later. In the present case, i.e., for an isothermal surface, we can use an arbitrary value for b^2/α, 1.0 being the simplest. But if a comparison of the results of this case is to be made with those for other cases, to be discussed later, it would be convenient to take the same value for b^2/α, so that the time and film thickness scales are the same. With the above definitions, we may now write Eq. (2.3) as:

$$\lambda = \sqrt{2(1 - r)\cdot \xi} \tag{2.9}$$

It is also important to consider the transient film condensation on an isothermal surface, for the second case when the flow is gravity dominated. A balance between the heat released in the condensation process and the energy conducted through the condensate film gives the mass rate of condensation as:

$$(\dot{m})_{in} = \frac{A\, k_1}{h_{fg}} \left[\frac{t_s - t_i}{\delta(\tau)} \right] \tag{2.10}$$

As the run-off is assumed to be gravity-dominated, viscous and surface tension effects being negligible, Bernoulli's equation may be applied. It can be shown that the mass flow rate through a differential area P·dy is proportional to the square root of the local height above the plate surface y, P being the perimeter of the plate allowing the flow of the condensate (refer Fig. 1). Therefore,

$$(d\dot{m})_{out} = \rho\sqrt{2gy} \cdot P\, dy \tag{2.11}$$

The total flow rate is found by integration to be:

$$(\dot{m})_{out} = \rho \sqrt{2g}\, P \int_{y=0}^{y=\delta(\tau)} \sqrt{y} \cdot dy$$

or,

$$(\dot{m})_{out} = \frac{2}{3} \sqrt{2g}\ \rho P\, [\delta(\tau)]^{3/2} \tag{2.12}$$

This is similar to the inviscid result for the flow over a wier and has been obtained by Mollendorf and Chu [24].

Considering the mass conservation, we write,

$$(\dot{m})_{in} - (\dot{m})_{out} = \rho A \frac{d\delta(\tau)}{d\tau} \qquad (2.13)$$

which, after use of Eqs. (2.10) and (2.12), gives,

$$\frac{k_1}{\rho h_{fg}} \frac{(t_s - t_i)}{\delta(\tau)} - \frac{2}{3}\sqrt{2g}\,\frac{P}{A}\,[\delta(\tau)]^{3/2} = \frac{d\delta(\tau)}{d\tau} \qquad (2.14)$$

Hence, using the dimensionless terms, from Eqs. (2.4) through (2.8), the above differential equation may be written as:

$$\lambda\dot{\lambda} + \gamma\,\lambda^{5/2} - 1 = 0 \qquad (2.15)$$

with the initial condition

$$\lambda = 0 \quad \text{at} \quad \xi = 0 \qquad (2.16)$$

where,

$$\dot{\lambda} = \frac{d\lambda}{d\xi}$$

and, γ is a dimensionless parameter for flow under gravity and is given by:

$$\gamma = \frac{2}{3}\sqrt{2g}\,\frac{P}{A}\,\frac{b^2}{\alpha}\left(\frac{b^2}{\alpha}\,\frac{k_1\,\theta_s}{\rho h_{fg}}\right)^{1/4} \qquad (2.17)$$

It must be noted that the γ is the only parameter that arises for flow under gravity, and $\gamma = 0$ represents the physical situation of no run-off. As γ is a strong function of P/A, whose value can vary greatly, γ can take a wide range of values. It will be equal to zero, for a physical circumstance, only when P is zero. It can, however, achieve

very small values, depending on fluid properties, the values of b, θ_s, α, etc. This is an important point which has not been brought out by Mollendorf and Chu [24] in their discussion on the flow parameter 'α', which is very similar to γ in the present case.

For the no flow condition, i.e., $\gamma = 0$, the differential equation (2.15), with the initial condition, Eq. (2.16), results in the same equation as that from Eq. (2.9) for $r = 0$. It should be noted that both $r = 0$ and $\gamma = 0$ represent the no flow condition.

As the expressions for heat flux for the transient film condensation on an isothermal surface are similar to those for other cases, to be discussed later, they have been presented separately in section (2.1.6).

2.1.2 Plate with Insulated Bottom Surface and a Negligible Temperature Variation Across its Thickness:

The physical situation being considered is shown schematically in Fig. 2. A horizontal plate of uniform thickness b with an insulated lower surface, is initially at a temperature t_i. At time $\tau = 0$, the top surface is subjected to a saturated vapour which is at a constant temperature t_s. It is further assumed that the Biot Number, $B_i = hb/k_p$ is small enough so that the temperature gradients in the plate are negligible and that it may be taken to be at a uniform temperature. Evidently, low values of the heat

transfer coefficient, h, and high values of the plate thermal conductivity, k_p, with sufficiently small plate thickness, b, will result in low values of Bi. This situation occurs very often in practice and physically corresponds to a negligible internal resistance to heat flow. In plates, the error introduced by the assumption that the temperature at any instant is uniform will be less than 5 percent when the internal resistance is less than 10 percent of the external surface resistance, i.e., when Bi < 0.1 [29]. It is also

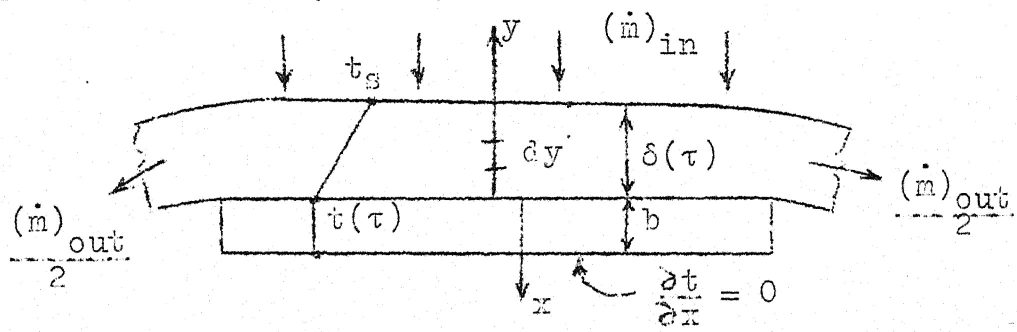

Fig. 2: Co-ordinate system for transient film condensation on a horizontal plate, with insulated bottom surface and a negligible temperature variation across its thickness.

assumed that the length and the width of the plate are sufficiently larger than the thickness so that the temperature gradients near the edge of the plate may be neglected.

Now we will consider the above situation for the earlier mentioned two circumstances of run-off. First, considering a specified fraction r, of the condensate, flowing out, we get the mass rate of condensation as:

$$(\dot{m})_{in} = \rho A \frac{d\delta(\tau)}{d\tau} / (1-r) \tag{2.18}$$

Now, equating the heat released due to condensation to the energy change in the plate, we obtain,

$$\frac{\rho\, h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} = \rho_p\, b\, C_{p_p} \frac{dt(\tau)}{d\tau} \tag{2.19}$$

Since the heat flux conducted through the condensate film is equal to the heat released in the condensation process, we get,

$$\frac{\rho\, h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} = k_1 \left[\frac{t_s - t(\tau)}{\delta(\tau)} \right] \tag{2.20}$$

Using excess temperature and dimensionless film thickness and time, defined earlier, in the above equation, we obtain,

$$d\lambda\ (\xi) = \frac{(1-r)}{\beta}\ d\phi\ (\xi) \tag{2.21}$$

where ϕ, the dimensionless temperature of the plate, is given by:

$$\phi = \frac{\theta}{\theta_s} = \frac{t - t_i}{t_s - t_i} \tag{2.22}$$

and, β the dimensionless condensation parameter is,

$$\beta = \sqrt{\frac{\rho\, h_{fg}\ \alpha\ k_1}{k_p^2\ \theta_s}} \tag{2.23}$$

Here, β is the most significant parameter, which controls condensation in the present case and in all other cases, to be discussed later.

Integrating the differential equation (2.21) and using the initial conditions that

$$\lambda = 0 \quad \text{and} \quad \phi = 0 \quad \text{at} \quad \xi = 0 \tag{2.24}$$

since $t = t_i$ at $\tau = 0$, we get,

$$\lambda = \frac{(1-r)}{\beta} \phi \tag{2.25}$$

Similarly, Eqs. (2.20) and (2.25) yield,

$$\dot{\lambda} = \frac{(1-r)}{\lambda} - \beta \tag{2.26}$$

Integrating Eq. (2.26) with the initial conditions, Eq.(2.24), and making use of Eq. (2.25), we get,

$$(1 - \phi)\, e^{\phi + \frac{\beta^2 \xi}{(1-r)}} = 1 \tag{2.27}$$

Now, we will consider the flow dominated by gravity. The heat released by condensation is equated to the energy conducted through the condensate film. This gives the mass rate of condensation as:

$$(\dot{m})_{in} = \frac{A\, k_1}{h_{fg}} \left[\frac{t_s - t(\tau)}{\delta(\tau)} \right] \tag{2.28}$$

The introduction of Eqs. (2.28) and (2.12) into Eq. (2.13) results in:

$$\phi = 1 - \lambda\, \dot{\lambda} - \gamma\, \lambda^{5/2} \tag{2.29}$$

A balance between the heat conducted through the condensate film to the energy change in the plate, yields:

$$k_1 \left[\frac{t_s - t(\tau)}{\delta(\tau)} \right] = \rho_p \, b \, C_{p_p} \frac{dt(\tau)}{d\tau} \tag{2.30}$$

In terms of dimensionless variables, Eqs. (2.29) and (2.30) gives,

$$\lambda\ddot{\lambda} + (\dot{\lambda} + \frac{5}{2}\gamma \lambda^{3/2} + \beta)\dot{\lambda} + \beta\gamma \lambda^{3/2} = 0 \tag{2.31}$$

with the initial conditions,

$$\lambda = 0 \quad \text{at} \quad \xi = 0 \tag{2.32}$$

The value of $\dot{\lambda}(0)$ is infinite, as can be seen from Eq.(2.26)

For the no flow condition, $\gamma = 0$, Eqs. (2.31) and (2.29) may be integrated. The result is the same as that obtained from Eqs. (2.25) and (2.27) for r = 0. The relations for no flow are:

$$\lambda = \frac{1}{\beta} \phi \tag{2.33}$$

and,
$$(1 - \phi)\, e^{\phi + \beta^2 \xi} = 1 \tag{2.34}$$

It should be noted that all the above results can be used for a plate with varying thickness along its length, provided that the top surface is horizontal and that at any place Bi is not greater than 0.1. The only modification required is that b must be taken as V/A, where V is the volume of the plate and A the area of the top surface.

2.1.3 Semi-infinite Approximation - Solution for Short Times and Thick Plates:

In this section, the film condensation over a horizontal plate of uniform thickness, initially at temperature t_i and behaving as a semi-infinite body, will be considered. It is assumed that the heat transfer effects have not penetrated beyond the penetration depth Δ, i.e., the plate temperature at and beyond Δ is t_i, the initial temperature. The plate is of a finite thickness b and, for short times, it behaves as a semi-infinite body.

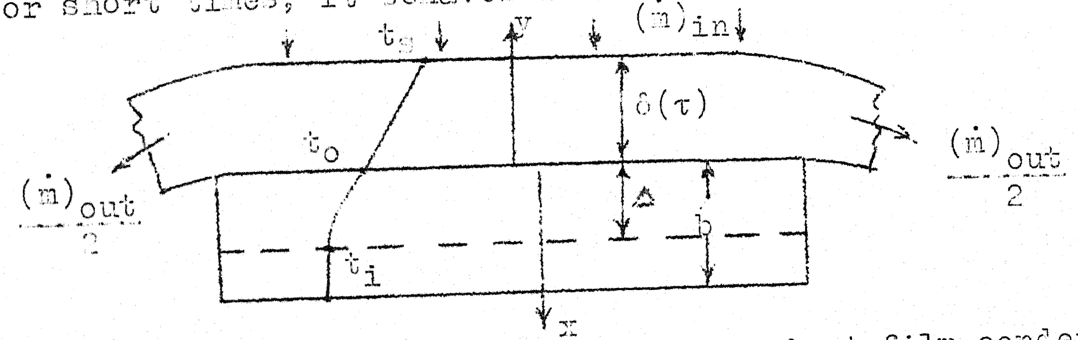

Fig. 3: Schematic of the transient film condensation, for short times, on a horizontal plate, of uniform thickness and behaving as a semi-infinite body.

The semi-infinite heat conduction problem is solved by the use of integral methods for nonlinear heat transfer developed by Goodman [27, 28]. A third order polynomial is employed for the temperature distribution in the plate.

For determining the four constants of a cubic profile, given as:

$$t - t_i = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

the following conditions are employed.

$$\phi\,(\overline{\Delta}\,(\xi),\ \xi) = 0 \tag{2.35a}$$

$$\frac{\partial\phi}{\partial X}\,(\overline{\Delta}\,,\ \xi\,) = 0 \tag{2.35b}$$

$$\frac{\partial\phi}{\partial X}\,(\,0,\ \xi\,) = -\,f\,(\phi_o,\ \xi\,) \tag{2.35c}$$

$$\frac{\partial^2\phi}{\partial X^2}\,(\overline{\Delta}\,,\ \xi\,) = 0 \tag{2.35d}$$

where X and $\overline{\Delta}$ are the dimensionless distance from the top surface and the dimensionless penetration depth, respectively. They have been non-dimensionalized with b as the characteristic dimension. Therefore,

$$X = x/b \tag{2.36}$$

and, $$\overline{\Delta} = \Delta/b \tag{2.37}$$

$\phi_o\,(\xi)$ is the dimensionless temperature of the upper surface of the plate and $f(\phi_o,\ \xi\,)$ is the heat flux, divided by thermal conductivity of the plate material.

The first three conditions (2.35a,b,c), known as natural conditions, arise from the considerations of the penetration of thermal effects in the plate and the boundary conditions at the top surface. The fourth condition is known as smoothening condition and is obtained in the following manner. Conversion of heat conduction equation,

$$\frac{\partial t}{\partial \tau} = \alpha\,\frac{\partial^2 t}{\partial x^2} \tag{2.38}$$

in terms of non-dimensional variables, yields,

$$\frac{\partial \phi}{\partial \xi} = \frac{\partial^2 \phi}{\partial X^2} \tag{2.39}$$

Now, by differentiating Eq. (2.35a) with respect to ξ and applying Eq. (2.39), the condition given in Eq. (2.35d) can be arrived [28].

The appropriate cubic temperature profile is, therefore, obtained as:

$$\phi = \phi_o \left[1 - \frac{X}{\bar{\Delta}} \right]^3 \tag{2.40}$$

where,

$$\bar{\Delta} = \frac{3\, \phi_o(\xi)}{f(\phi_o, \xi)} \tag{2.41}$$

Integrating heat conduction equation (2.39) with respect to X, as done by Goodman [27, 28], we obtain:

$$f\,(\phi_o,\ \xi) = \frac{d}{d\xi} \left[\frac{3}{4} \frac{\phi_o^{\,2}(\xi)}{f\,(\phi_o,\ \xi)} \right] \tag{2.42}$$

or, $$\frac{4}{3} f^2 = 2\, \phi_o \frac{d\phi_o}{d\xi} - \frac{\phi_o^{\,2}}{f} \cdot \frac{df}{d\xi} \tag{2.43}$$

Now, first, we will consider a specified fraction of the condensate flowing out of the edges of the plate. A balance between the heat flux being conducted through the condensate film and the heat released in condensation of the vapour, results in:

$$\frac{\rho h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} = k_1 \left[\frac{t_s - t_o(\tau)}{\delta(\tau)} \right] \tag{2.44}$$

Non-dimensionalization yields an equation of the form:

$$\phi_o = 1 - \frac{\lambda\dot{\lambda}}{(1-r)} \tag{2.45}$$

As the heat flux conducted into the plate is same as the heat released in condensation, we obtain,

$$\frac{\rho h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} = - k_p \frac{\partial t}{\partial x} (0, \tau) \tag{2.46}$$

which, in its non-dimensional form, gives,

$$\frac{\partial \phi}{\partial X} (0, \xi) = - \frac{\beta}{(1-r)} \dot{\lambda} \tag{2.47}$$

From Eqs. (2.35c) and (2.47), we get,

$$f(\phi_o, \xi) = \frac{\beta}{(1-r)} \dot{\lambda} \tag{2.48}$$

The introduction of Eqs. (2.45) and (2.48) into the Eq.(2.43) yields:

$$[(1-r)^2 - \lambda^2 \dot{\lambda}^2] \ddot{\lambda} + [2 (1-r) + \frac{4}{5} \beta^2 - 2 \lambda\dot{\lambda}] \dot{\lambda}^3 = 0 \tag{2.49}$$

From the above equation it can be shown that $\lambda\dot{\lambda}$ must be a constant, since r and β are constants and $\dot{\lambda}$ and $\ddot{\lambda}$ vary with time. Let,

$$\lambda\dot{\lambda} = \text{Constant} = N_1 \tag{2.50}$$

then from Eq. (2.49),

$$\ddot{\lambda} = - Q \lambda^3 \tag{2.51}$$

where,

$$Q = - \left[\frac{2N_1 - 2(1-r) - \frac{4}{3}\beta^2}{(1-r)^2 - N_1^2} \right] \tag{2.52}$$

Integrating the differential equation (2.51) and using the initial condition, Eq. (2.32), which is valid in the present case also, we get,

$$\lambda = \sqrt{\frac{2}{Q}\xi} \tag{2.53}$$

Hence,

$$N_1 = \frac{1}{Q} \tag{2.54}$$

Using Eqs. (2.52) and (2.54), we get the quadratic equation,

$$N_1^2 - \left[2(1-r) + \frac{4}{3}\beta^2\right] N_1 + (1-r)^2 = 0 \tag{2.55}$$

whose roots are given by,

$$N_1 = (1-r) + \frac{2}{3}\beta^2 \pm \sqrt{\frac{4}{3}(1-r)\beta^2 + \frac{4}{9}\beta^4} \tag{2.56}$$

Hence, the solution for Eq. (2.50) and, thus, for the differential equation (2.49) is given by,

$$\lambda = N \xi^{1/2} \tag{2.57}$$

where,

$$N = \sqrt{2N_1} = \left[2(1-r) + \frac{4}{3}\beta^2 \pm \sqrt{\frac{16}{3}(1-r)\beta^2 + \frac{16}{9}\beta^4}\right]^{1/2} \tag{2.58}$$

Since, the rate of increase in the condensate film thickness should not be more than that over an isothermal surface at the initial temperature t_i, because of the plate thermal resistance introduced in the present case, we obtain the following condition from Eq. (2.9),

$$N < \sqrt{2(1-r)}$$

Hence, the accepted value for N is,

$$N = [2(1-r) + \frac{4}{3}\beta^2 - \sqrt{\frac{16}{3}(1-r)\beta^2 + \frac{16}{9}\beta^4}\ ]^{1/2} \tag{2.59}$$

From the Eqs. (2.41), (2.45), (2.48) and (2.57) the non-dimensional penetration depth is obtained as:

$$\bar{\Delta} = M\,\xi^{1/2} \tag{2.60}$$

where,

$$M = \frac{3[2(1-r) - N^2]}{\beta N} \tag{2.61}$$

Eqs. (2.45) and (2.57) give the dimensionless temperature of the top surface of the plate as:

$$\phi_o = 1 - \frac{N^2}{2(1-r)} \tag{2.62}$$

From Eqs. (2.40), (2.60) and (2.62) the temperature in the plate at any distance X is obtained as:

$$\phi = [\,1 - \frac{N^2}{2(1-r)}\,]\,(1 - \frac{X}{M}\,\xi^{-1/2})^3 \tag{2.63}$$

The equation (2.62) is quite significant and shows that the temperature of the top surface of the plate will attain a constant value immediately after it is exposed to the saturated vapour, till the plate behaves as a semi-infinite body. This temperature is a function of r and β. For the no flow condition, it is a function of β only.

For the flow dominated by gravity, the mass rate of condensation is obtained as:

$$(\dot{m})_{in} = \frac{A\ k_1}{h_{fg}} \left[\frac{t_s - t_o(\tau)}{\delta(\tau)} \right] \tag{2.64}$$

From Eqs. (2.12) and (2.13), which are applicable in the present case also, and Eq.(2.64) ϕ_o is obtained in terms of λ and γ as:

$$\phi_o = 1 - \lambda\dot{\lambda} - \gamma\,\lambda^{5/2} \tag{2.65}$$

Again, equating the heat flux conducted through the condensate film to the heat conducted into the plate at x = 0, we get,

$$k_1 \left[\frac{t_s - t_o(\tau)}{\delta(\tau)} \right] = - k_p \frac{\partial t}{\partial x} (0, \tau) \tag{2.66}$$

Expressing the above equation in terms of dimensionless variables and employing Eqs. (2.65) and (2.35c), we get,

$$f(\phi_o, \xi) = \beta\,(\dot{\lambda} + \gamma\,\lambda^{3/2}) \tag{2.67}$$

The introduction of Eqs. (2.65) and (2.67) into the Eq.(2.43), which is basically a conduction equation for the plate and is, therefore, applicable here also, results in:

$$[1-(\lambda\dot{\lambda}+\gamma\,\lambda^{5/2})^2]\ddot{\lambda} + \frac{4}{3}\beta^2(\dot{\lambda}+\gamma\,\lambda^{3/2})^3$$
$$+ (1-\lambda\dot{\lambda}-\gamma\,\lambda^{5/2})\,(2\dot{\lambda}^3+\frac{11}{2}\gamma\,\lambda^{3/2}\,\dot{\lambda}^2$$
$$+\frac{7}{2}\gamma^2\,\lambda^3\,\dot{\lambda}+\frac{3}{2}\gamma\,\lambda^{1/2}\,\dot{\lambda}) = 0 \qquad (2.68)$$

with the initial condition given by Eq. (2.32). The penetration depth is, thus, given by Eqs. (2.41), (2.65) and (2.67) as:

$$\overline{\Delta} = \frac{3}{\beta}\left[\frac{1}{(\dot{\lambda}+\gamma\,\lambda^{3/2})} - \lambda\right] \qquad (2.69)$$

The temperature at any distance X is obtained from Eqs.(2.40) and (2.69) as:

$$\phi = \phi_o\left[1-\frac{\beta}{3}\,\frac{(\dot{\lambda}+\gamma\,\lambda^{3/2})}{(1-\lambda\dot{\lambda}-\gamma\,\lambda^{5/2})}\,X\right]^3 \qquad (2.70)$$

Again, for the no flow condition, $\gamma = 0$, Eq. (2.68) yields the same expression as that obtained from Eq.(2.49) for $r = 0$.

2.1.4 Plate of Finite Thickness and Bottom Surface at a Fixed Temperature:

The transient film condensation on a horizontal plate, with finite but uniform thickness, and whose surface is maintained at the initial temperature t_i is considered in this section. The solution of this problem will follow the previous solution, in which the plate had been considered as semi-infinite. Hence, the finite thickness of the plate will affect the solution beyond a time interval given by the time at which the penetration depth equals the depth b

for the semi-infinite solution. The physical situation is as shown in Fig. 4.

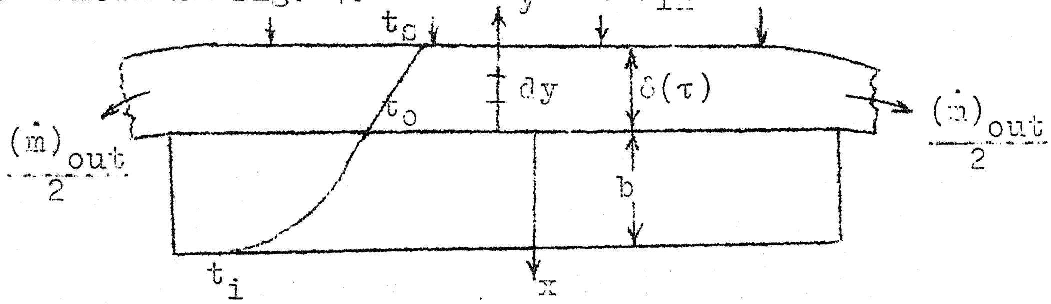

Fig. 4 Physical Model for transient film condensation flow on a horizontal plate, of finite thickness and bottom surface at a fixed temperature.

Again, for the present problem, the integral method developed by Goodman [27], is employed. A third order polynomial for the temperature distribution in the plate is assumed. For determining this cubic profile, of the temperature in the plate, the required four conditions are:

$$\phi\ (0,\ \xi) = \phi_o\ (\xi) \tag{2.71a}$$

$$\frac{\partial \phi}{\partial X}\ (0, \xi) = -f\ (\phi_o,\ \xi) \tag{2.71b}$$

$$\phi\ (1, \xi) = 0 \tag{2.71c}$$

$$\frac{\partial^2 \phi}{\partial X^2}\ (1, \xi) = 0 \tag{2.71d}$$

The first three conditions are derived from the boundary conditions. As the temperature of the bottom surface is fixed and does not change with respect to time, we obtain,

$$\left.\frac{\partial \phi}{\partial \xi}\right|_{X=1} = 0 \tag{2.72}$$

and, hence, from the heat conduction equation (2.39), the fourth condition, given by Eq. (2.71d), is obtained.

The appropriate cubic temperature profile is, therefore, given by [27], as:

$$\phi(X, \xi) = \left[\frac{3}{2} \phi_o(\xi) - \frac{1}{2} f(\phi_o, \xi) \right] (1 - X) + \frac{1}{2} \left[f(\phi_o, \xi) - \phi_o \right] (1 - X)^3 \tag{2.73}$$

Integrating the heat conduction equation (2.39) with respect to X and applying the necessary conditions as done by Goodman [27], we obtain,

$$12 \left[f(\phi_o, \xi) - \phi_o(\xi) \right] = 5 \frac{d\phi_o(\xi)}{d\xi} - \frac{df(\phi_o, \xi)}{d\xi} \tag{2.74}$$

For the first type of flow, i.e., a specified fraction of the condensate flowing out across the edges of the plate, the Eqs. (2.45) and (2.48) are applicable, and an introduction of these equations into Eq. (2.74) results in:

$$(\beta + 5\lambda)\ddot{\lambda} + (5\dot{\lambda} + 12\lambda + 12\beta)\dot{\lambda} - 12(1-r) = 0 \tag{2.75}$$

The initial conditions for the above equation are to be obtained from the semi-infinite solution, Eq. (2.57). Values of λ and $\dot{\lambda}$ at the end of the semi-infinite solution, i.e., for $\tau = \tau_s$, when $\Delta = b$, are used as the initial values for the above equation. Hence the initial conditions are:

$$\lambda = \lambda_s, \quad \dot{\lambda} = \dot{\lambda}_s \quad \text{at} \quad \xi = \xi_s \quad \text{when} \quad \bar{\Delta} = 1 \tag{2.76}$$

The temperature at any distance X is given by Eqs. (2.45), (2.48) and (2.73) as:

$$\phi = \frac{(1 - X)}{(1 - r)} \left[\{ (1-r) - \lambda\dot{\lambda} \} \left(1 + X - \frac{X^2}{2}\right) - \beta \dot{\lambda} \left(X - \frac{X^2}{2}\right) \right] \tag{2.77}$$

For flow under gravity, Eqs. (2.65), and (2.67), of section (2.1.3), are applicable. Introducing these equations into Eq. (2.74), we obtain,

$$(\beta + 5\lambda)\ddot{\lambda} + \left(5\dot{\lambda} + \frac{25}{2}\gamma\lambda^{3/2} + 12\lambda + \frac{3}{2}\beta\gamma\lambda^{1/2} + 12\beta\right)\dot{\lambda} + 12\gamma(\beta + \lambda)\lambda^{3/2} - 12 = 0 \tag{2.78}$$

with the initial conditions given by Eq. (2.76). The values of λ_s, $\dot{\lambda}_s$ at ξ_s, in the present case, are obtained from Eq. (2.68). For the no flow condition, Eqs. (2.78) and (2.75) are identical. The temperature in the plate is obtained by the substitution of Eqs. (2.65) and (2.67) into Eq. (2.73) as:

$$\phi = (1 - X)\left[(1 - \lambda\dot{\lambda} - \gamma\lambda^{5/2}) \left(1 + X - \frac{X^2}{2}\right) - \beta(\dot{\lambda} + \gamma\lambda^{3/2}) \left(X - \frac{X^2}{2}\right) \right] \tag{2.79}$$

2.1.5 Plate of Finite Thickness and Insulated Bottom Surface:

The physical situation to be considered is as shown in Fig. 5. A horizontal plate of uniform thickness b, with an insulated lower surface is initially at a temperature t_i. At time $\tau = 0$, the top surface is exposed to a saturated vapour, maintained at a constant temperature t_s. Initially,

the plate behaves as a semi-infinite body in the x-direction. After a time τ_s, when the penetration depth Δ equals b, the finite thickness of the plate will affect the solution. A solution will be developed for time $\tau > \tau_s$. This problem would also arise when a plate of finite thickness is immersed in a vapour so that condensation occurs on both sides, the midplane being adiabatic for symmetric heat transfer conditions on the two faces.

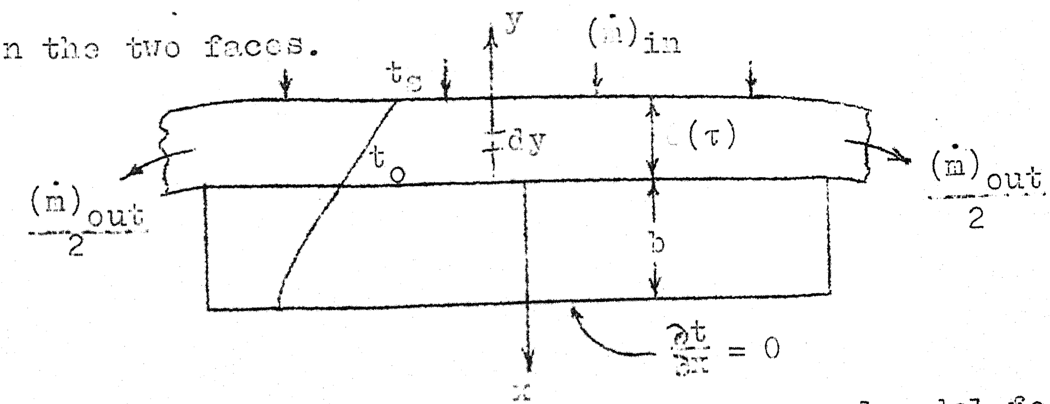

Fig. 5: Co-ordinate system and physical model for the transient film condensation on a plate of finite thickness and insulated bottom surface.

Again, for the present problem, the integral method, discussed earlier, for the non-linear heat transfer problem is employed. As only three boundary conditions for the temperature in the plate are known, namely,

$$\phi(0, \xi) = \phi_o(\xi) \tag{2.80a}$$

$$\frac{\partial \phi}{\partial x}(0, \xi) = -f(\phi_o, \xi) \tag{2.80b}$$

$$\frac{\partial \phi}{\partial x}(1, \xi) = 0 \tag{2.80c}$$

We can employ only a second order profile. The appropriate quadratic temperature profile for the plate is:

$$\phi = [\,\phi_o(\xi) - \tfrac{1}{2} f(\phi_o, \xi\,)] + \tfrac{1}{2} f\,(\phi_o, \xi)\,(1 - X)^2 \tag{2.81}$$

Integrating the heat conduction equation (2.39) with respect to X and applying the required conditions, we get,

$$\frac{d}{d\xi}\,[\int_0^1 \phi .\, dx] = \frac{\partial \phi}{\partial X}(1, \xi) - \frac{\partial \phi}{\partial X}(0, \xi)$$

which, after substitution of Eqs. (2.80c) and (2.81), results in:

$$f(\phi_o, \xi\,) = \frac{d}{d\xi}\,[\phi_o(\xi) - \tfrac{1}{3}\; f(\phi_o, \xi)] \tag{2.82}$$

or,

$$f(\phi_o,\; \xi) = \frac{d\;\phi_o(\xi)}{d\xi} - \frac{1}{3}\;\frac{df\;(\phi_o,\xi)}{d\xi} \tag{2.83}$$

The Eq. (2.82) is basically a heat conduction equation and can be used for the purposes other than the present one, for example, for nonlinear heat transfer, in a slab of finite thickness and one side insulated, with the temperature of the other surface a function of time and the heat flux a prescribed function of surface temperature and time.

For the flow situation, when a specified fraction of the condensate is flowing out across the edges of the plate, Eqs. (2.45) and (2.48) are applicable and use of these equations alongwith Eq. (2.83) results in:

$$(\lambda + \frac{\beta}{3}\,)\,\ddot{\lambda} + (\,\dot{\lambda} + \beta\,)\,\dot{\lambda} = 0 \tag{2.84}$$

with the initial conditions given by Eq. (2.76). The values of λ_s and $\dot{\lambda}_s$ at $\xi = \xi_s$ are to be obtained from Eq.(2.57).

Apparently, the above equation does not involve the flow term r. But the value of r affects the solution of this equation, because the solution has to follow the semi-infinite results, which depend on r, as shown earlier. The dimensionless temperature is given by Eq. (2.81) after employing Eqs. (2.45) and (2.48) as:

$$\emptyset = 1 - [\lambda - \beta (X - \frac{X^2}{2})] \frac{\dot{\lambda}}{(1-r)} \qquad (2.85)$$

For the flow under gravity, Eqs. (2.65) and (2.67) of section (2.1.3), are, again, applicable. Hence these equations with Eq. (2.83) gives,

$$(\frac{\beta}{3} + \lambda) \ddot{\lambda} + (\dot{\lambda} + \frac{5}{2} \gamma \lambda^{3/2} + \frac{1}{2} \beta \gamma \lambda^{1/2} + \beta) \dot{\lambda} + \beta \gamma \lambda^{3/2} = 0 \qquad (2.86)$$

with the initial conditions given by Eq. (2.76). Here, the values of λ_s and $\dot{\lambda}_s$ at $\xi = \xi_s$ are given by Eq. (2.68). From Eqs. (2.65), (2.67) and (2.81), we get the dimensionless temperature in the plate as:

$$\emptyset = 1 - [\lambda + \beta (X - \frac{X^2}{X})] (\dot{\lambda} + \gamma \lambda^{3/2}) \qquad (2.87)$$

2.1.6 Heat Flux:

Since the equations of heat transfer, for the transient film condensation on a horizontal body, with various boundary conditions discussed earlier, are similar, except for a change due to the type of flow, they can be presented at one place.

For a specified fraction of the rate of condensation, flowing out across the edges of the condensing surface, the heat flux released due to condensation of the vapour is given by:

$$q = \frac{\rho\, h_{fg}}{(1-r)} \frac{d\delta(\tau)}{d\tau} \qquad (2.88)$$

which, in non-dimensional form, gives,

$$q = \frac{k_p\, \theta_s}{b} \frac{\beta}{(1-r)} \dot{\lambda} \qquad (2.89)$$

We may, for simplification, introduce a heat flux parameter, $\bar{q}$ given as:

$$\bar{q} = \frac{q}{\left(\frac{k_p\, \theta_s}{b}\right)} = \frac{\beta}{(1-r)} \dot{\lambda} \qquad (2.90)$$

The above equations are valid for all the five different boundary conditions, discussed earlier.

For flow under gravity, the heat flux conducted through the condensate film is given by:

$$q = k_l \left[\frac{t_s - t_o}{\delta} \right] \qquad (2.91)$$

which, in terms of dimensionless variables, gives,

$$q = \frac{k_p \theta_s}{b} \beta (\dot{\lambda} + \gamma \lambda^{3/2}) \tag{2.92}$$

Hence, the heat flux parameter $\bar{q}$ is obtained as:

$$\bar{q} = \frac{q}{\left(\frac{k_p \theta_s}{b}\right)} = \beta (\dot{\lambda} + \gamma \lambda^{3/2}) \tag{2.93}$$

It should be noted that the top surface temperature t_o in Eq. (2.91) is equal to t_i in case of condensation on isothermal surface and to $t(\tau)$ for condensation over a plate with bottom surface insulated and temperature variation negligible. Again, in this case, the above equations are valid for the various boundary conditions, for the plate, discussed earlier, provided the flow is gravity dominated. The heat flux parameter $\bar{q}$, for the no flow condition, is, therefore, given by,

$$\bar{q} = \beta \dot{\lambda} \tag{2.94}$$

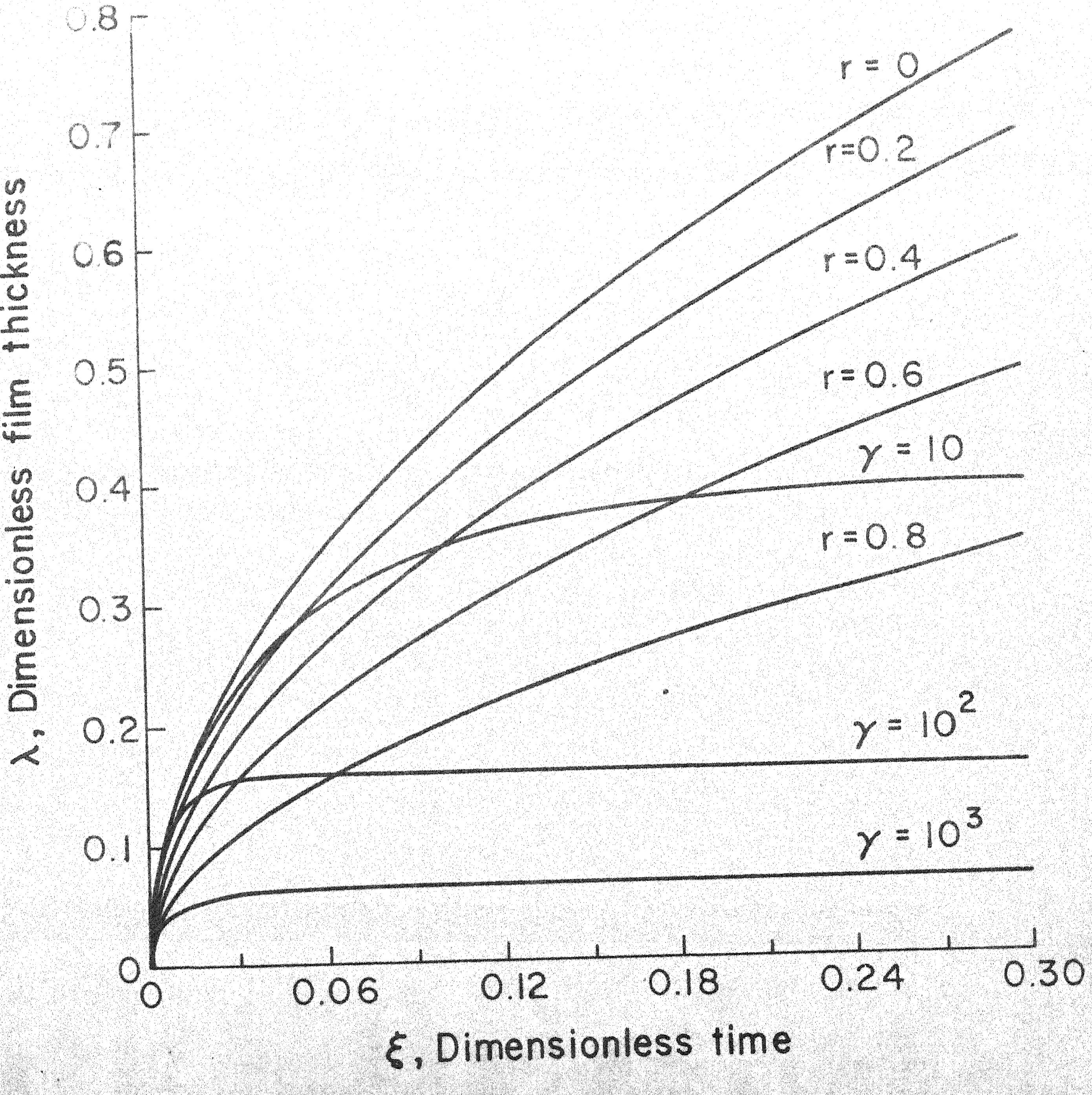

Fig. 6 Effect of fractional flow parameter r and gravity flow parameter γ on dimensionless film thickness λ, for transient film-condensation on a horizontal isothermal surface.

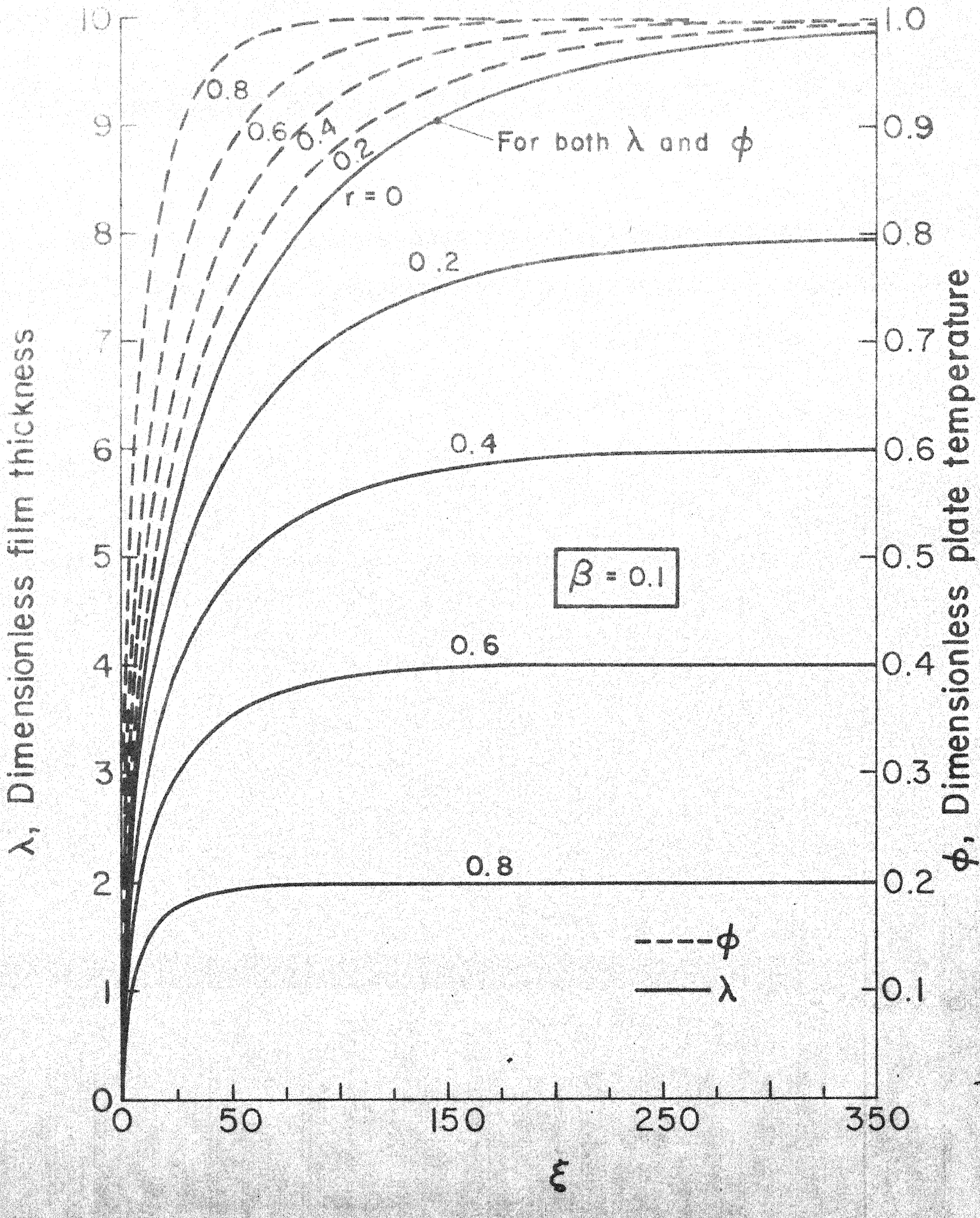

Fig. 7 Effect of r on λ and φ for transient film-condensation on a horizontal plate, with bottom surface insulated and temperature variation across its thickness negligible.

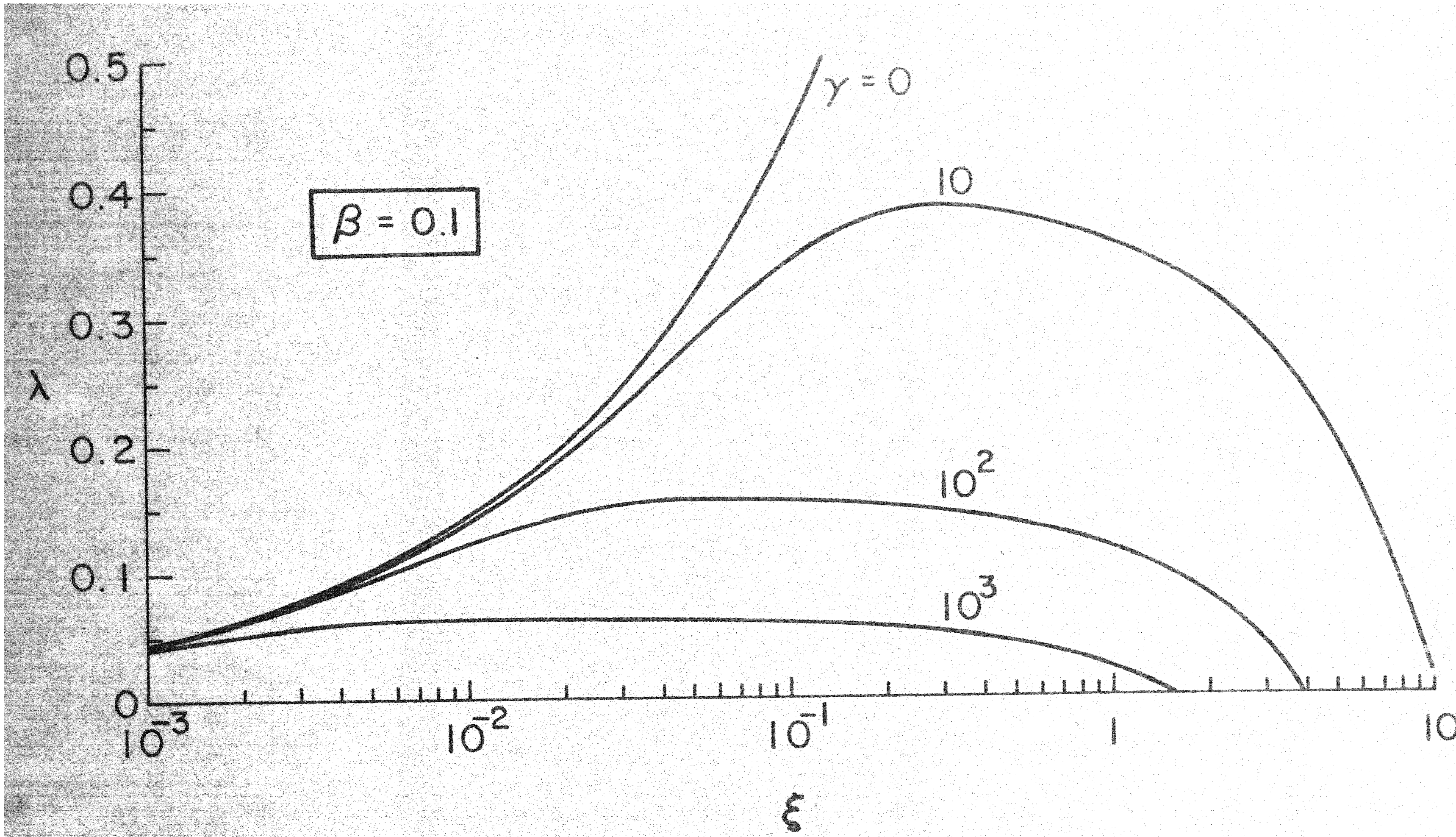

Fig. 8 Dependence of λ on γ for transient film-condensation on a horizontal plate, with bottom surface insulated and temperature variation across its thickness negligible.

2.2 Results and Discussion:

(a) Isothermal Surface: Figure 6 presents the variation of the dimensionless film thickness λ with respect to dimensionless time ξ, for various values of r and γ. While for any value of r, which must obviously be less than 1.0, the film thickness λ increases with time, it achieves a constant value after a certain time for $\gamma > 0$. The time, for λ to become constant, is found to decrease as the value of γ increases. For the no flow condition, $r = 0$, Eq. (2.3) is the same as the expression obtained by Gebhart [19], and by Rohsenow and Hartnett [23b]. For small values of $C_{p_1} \Delta t/h_{fg}$ the results given by Eq. (2.3), or by Eq. (2.9), are in good agreement with the exact results obtained by Pfahl [22] for the no flow condition. The error is only 1.5 percent when $C_{p_1} \Delta t/h_{fg}$ equals 0.135. The equations (2.3) and (2.9) are also much simpler than the expressions given by Pfahl [22].

(b) Plate with Insulated Bottom Surface and Negligible Temperature Variation Across Its Thickness:

The calculated dimensionless film thickness λ and the dimensionless plate temperature ϕ are shown in Figs. 7-9, for a range of r and γ. Increasing r or γ is seen to decrease the maximum film thickness and to increase the rate of temperature rise of the plate. This is reasonable since a thinner film is expected for greater flow, obviously, offering less resistance to heat flow. In each case the plate temperature

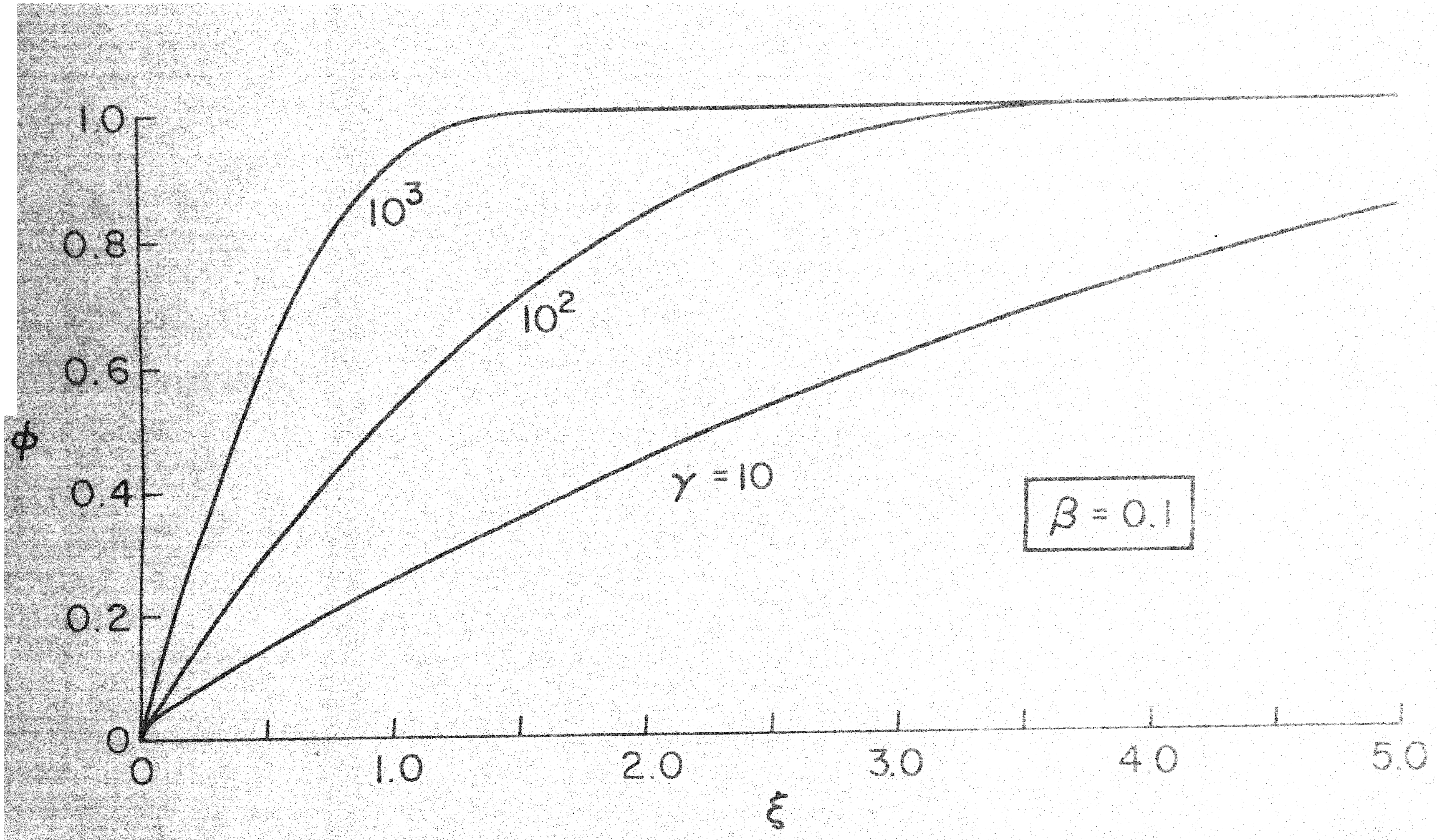

Fig.9 Effect of γ on ϕ for transient film-condensation on a horizontal plate, with bottom surface insulated and temperature variation across its thickness negligible.

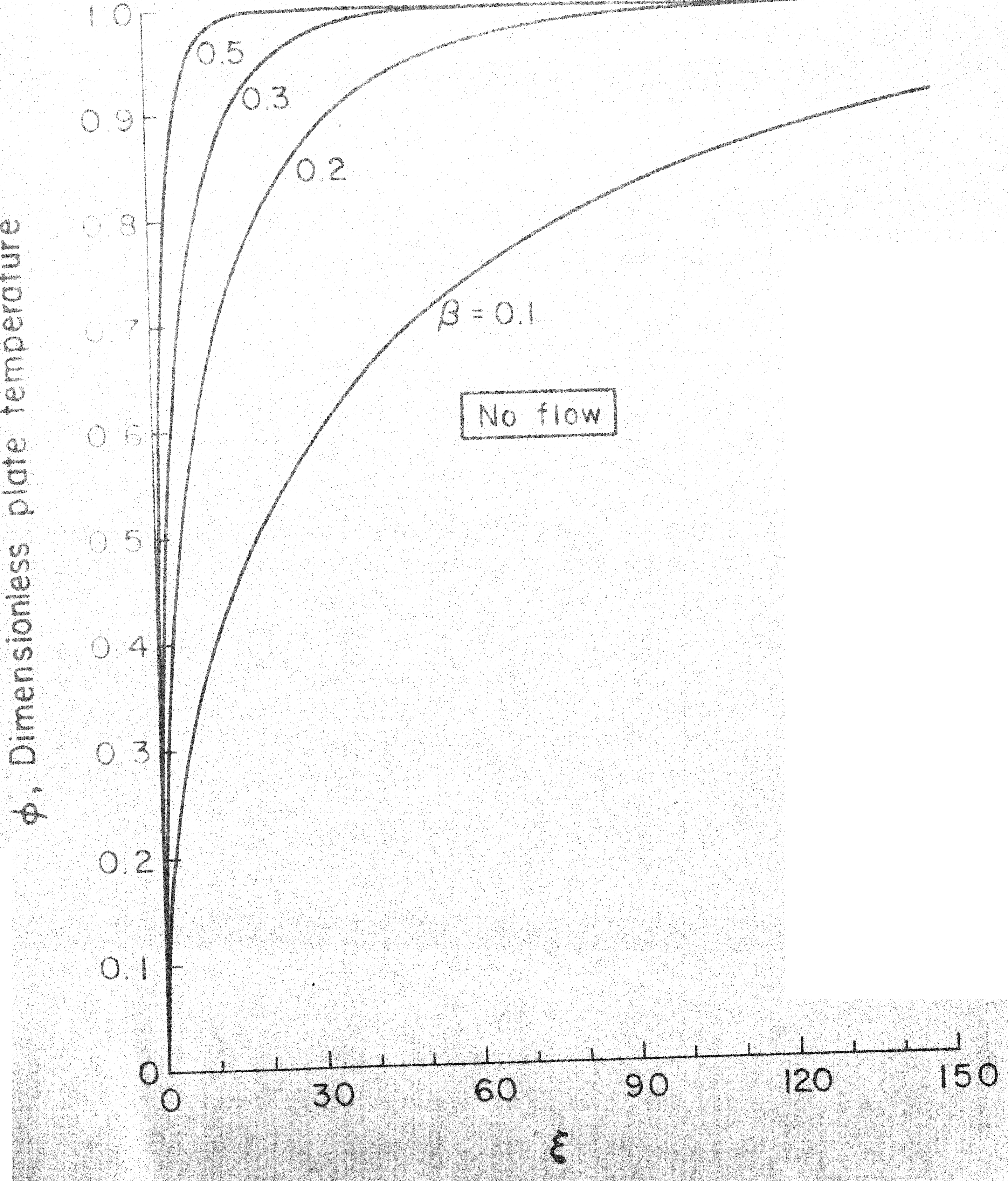

Fig. 10 Effect of condensation parameter β on ϕ for transient film-condensation on a horizontal plate, with bottom surface insulated and temperature variation across its thickness negligible.

reaches a value very close to saturation temperature of the vapour. While for any value of r, the film thickness is constant after a certain time, it becomes zero ultimately for $\gamma > 0$. This is expected since no condensation occurs when the body reaches the saturation temperature. Clearly, therefore, physically, the model based on gravity is more suitable for large times, since the model based on flow fraction r gives a constant value of λ. At low times, this latter model may be quite suitable, particularly, in practical situations. Mollendorf and Chu [24] have also obtained the expressions for the film thickness and the plate temperature and have presented the results for various values of the flow parameter 'α', which is similar to γ in our case. Their results are same as ours. They have also reported that for the no flow condition, the agreement between theory and experiment is excellent.

Figure 10 shows the effect of β, the dimensionless condensation parameter, on the film thickness for the no flow condition. The rate of heat transfer increases with increase in β, and, hence, the plate temperature t reaches a value close to the saturation temperature t_s much earlier. Obviously, the maximum film thickness is greatly reduced as β is increased. The same trend is obtained for all other cases, i.e., various boundary conditions assumed for the plate, and for all values of r and γ.

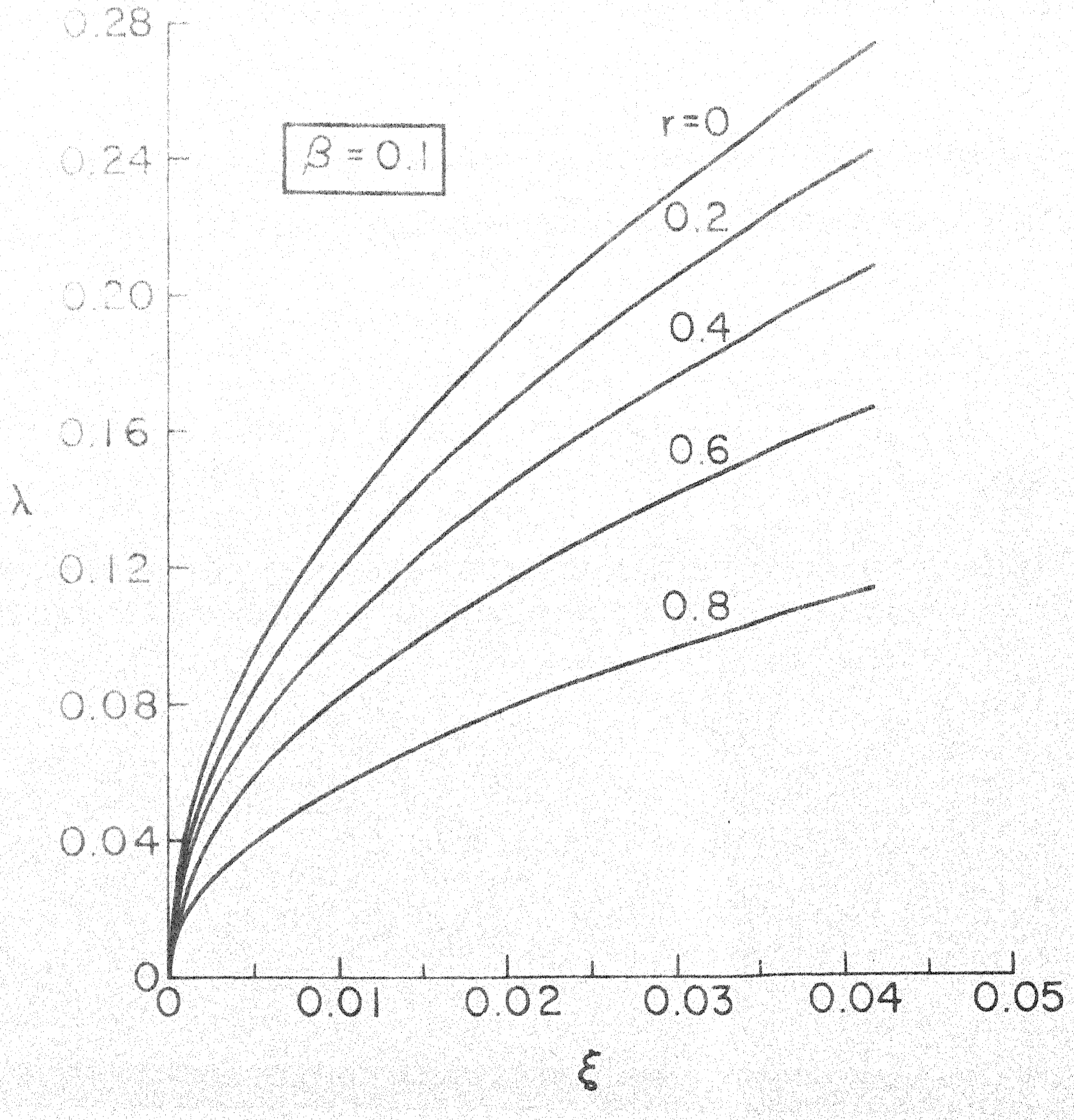

Fig. 11 Variation of λ with ξ, over a range of r, for transient film-condensation on a horizontal plate of uniform thickness, behaving as a semi-infinite body, for short times.

(c) Solution for Short Times and Thick Plates:

With an assumed third order temperature profile in the plate, the semi-infinite solution, for a fraction r of the condensate flowing out across the edges of the plate, Eq.(2.57) indicates that the film thickness is proportional to the square root of time, the proportionality constant being a function of r and β, i.e., of the fluid and the plate properties, Δt, flow fraction r etc. The constant of proportionality N, given by Eq. (2.59) decreases with an increase in r and β. Fig. 11 shows the film thickness for a range of r. For the no flow condition, Pfahl [22] has reported the same behaviour for the film thickness. There is a difference in the proportionality constant, the evaluation of which is quite difficult in his case and involves the error function. For small values of $C_{p_1} \Delta t/h_{fg}$ the results given by Eq. (2.57) are in good agreement with the exact results obtained by Pfahl [22]. For a saturated steam at atmospheric pressure condensing on a Ni-steel plate (Ni - 40 %), initially at 27°C, the results obtained in this study differs by only 0.1 percent from those obtained by Pfahl [22].

Interestingly, the temperature of the upper surface of the plate ϕ_o, given by Eq. (2.62), attains a constant value, depending upon r and β, as soon as it is exposed to a saturated vapour; and remains at that temperature as long as the effect of the bottom surface, on the solution, is not felt.

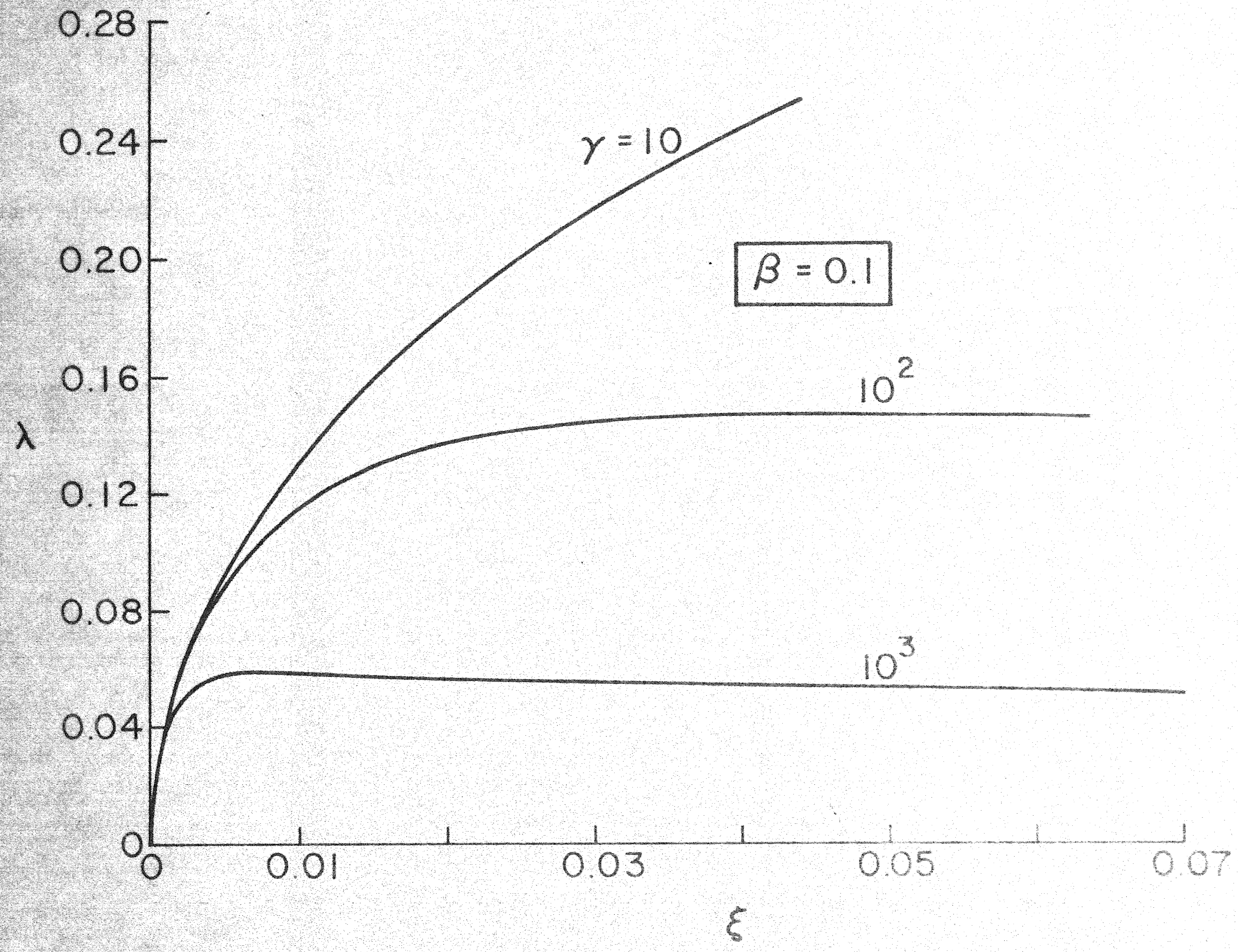

Fig. 12 Dependence of the transient response of λ on γ for film-condensation on a horizontal plate, behaving as a semi-infinite body.

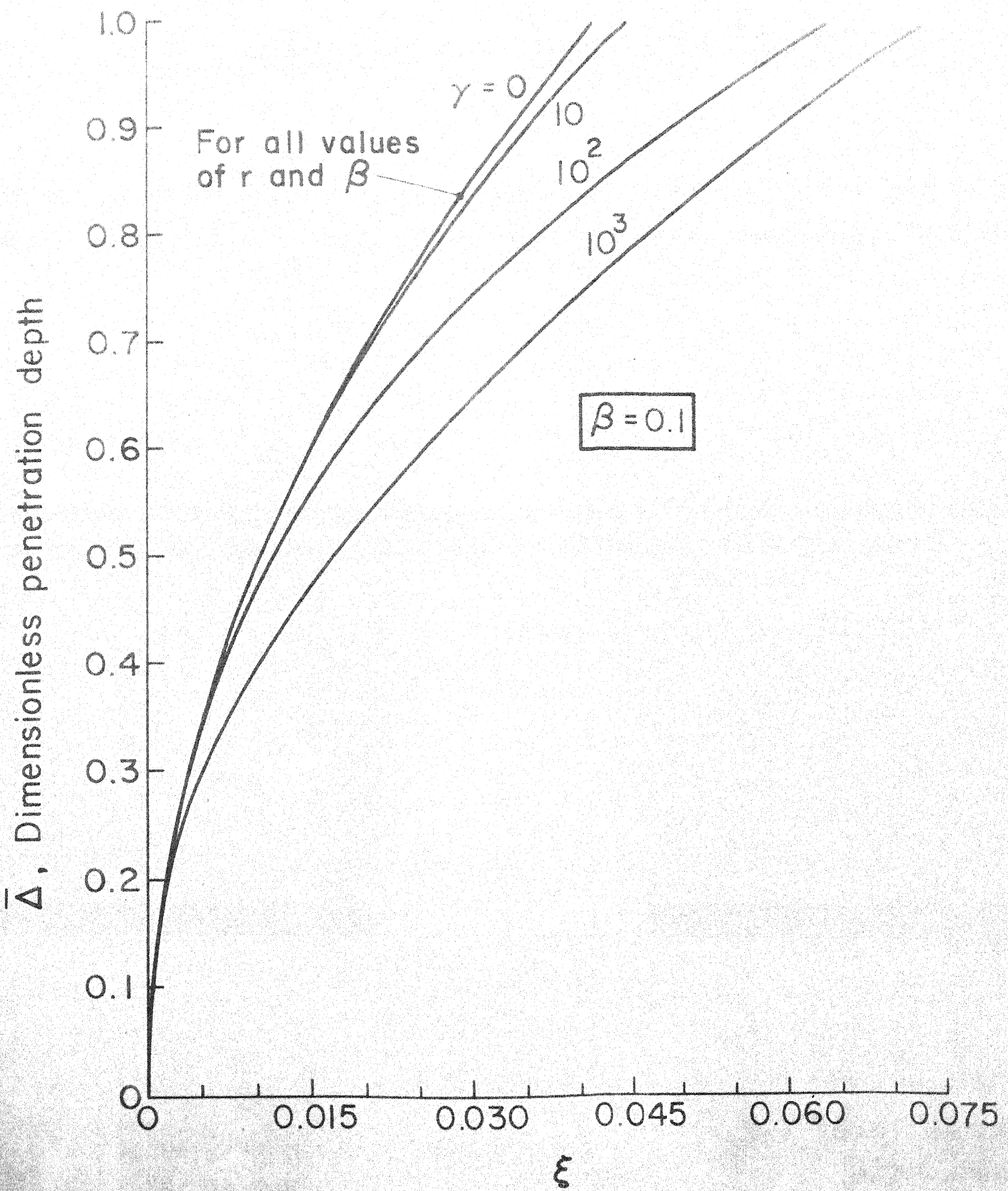

Fig. 13 Dependence of dimensionless penetration depth $\overline{\Delta}$ on r and γ for transient film-condensation on a horizontal plate behaving as a semi-infinite body.

The greater the values of r and β, the larger is the surface temperature ϕ_o.

For the flow dominated by gravity, the effect of γ over the film thickness is shown in Fig. 12. Here, the surface temperature ϕ_o is not a constant but increases with time. Greater the value of β or γ, greater is the rate of increase of ϕ_o.

For the no flow condition and a fraction r of the condensate flowing out across the edges of the plate, the penetration depth, given by Eq. (2.60), is proportional to the square root of time, as may be expected. The constant M in Eq. (2.60) is a function of r and β and is found to be the same for almost all combinations of r and β, and, hence, the penetration depth, for all values of r and β, is represented by only one curve, shown in Fig. 13. As the temperature of the upper surface of the plate t_o, in this case, attains a constant value as soon as it is exposed to the condensing vapour, it is a case of a step change from the initial temperature t_i to a constant temperature t_o. Using a third order temperature profile, Eckert and Drake [31] give the dimensionless penetration depth $\bar{\Delta}$ as $\sqrt{12\,\xi}$ for the above case. Goodman [27] has used a third order profile of a different form and has found $\bar{\Delta}$ as equal to $\sqrt{24\,\xi}$, which is expected to be a better result than the result obtained by Eckert and Drake due to the assumptions made. In the present study, M is within 1 percent of $\sqrt{24}$ for almost all combinations of r and β. Fig. 13 also

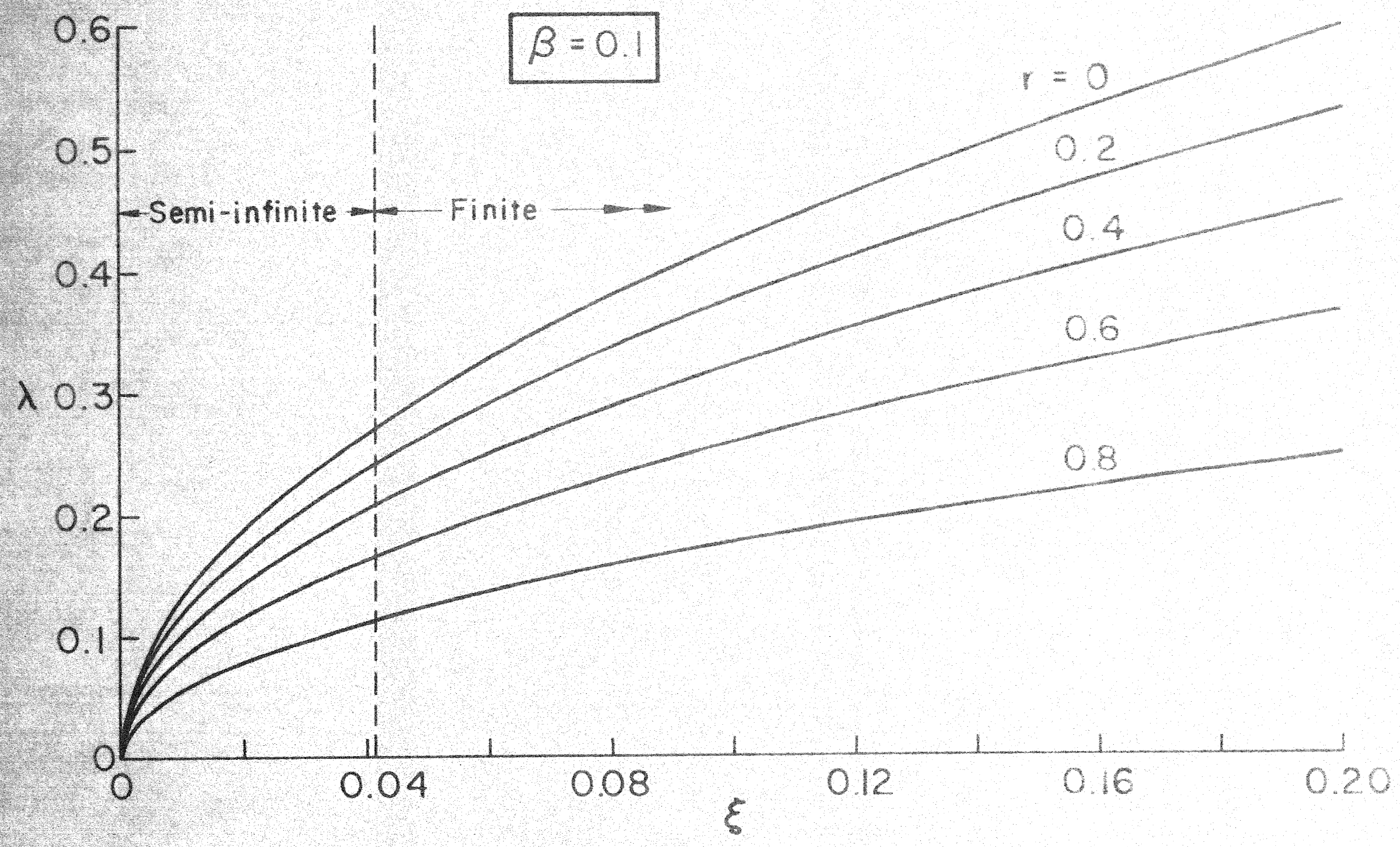

Fig. 14 Variation of λ with ξ, over a range of r, for transient film-condensation on a horizontal plate, of finite thickness and bottom surface at a fixed temperature.

shows the effect of the flow parameter γ on the penetration depth. It is observed that the rate of penetration decreases as γ increases.

(d) Plate of Finite Thickness and Bottom Surface at a Fixed Temperature:

Fig. 14 shows the effect of r on the film thickness, which is found to continue increasing with time. It is also observed that the switch over, from semi-infinite solution to the solution for finite thickness, is quite smooth. The deviation from the former is a gradual process. Fig. 15 presents the temperatures at different positions across the plate for the no flow condition. The upper surface temperature, which is constant as long as the plate behaves as a semi-infinite body, starts decreasing as soon as heat transfer occurs at the bottom surface, and after a very short time, starts increasing again till a maximum value is reached. Temperatures, at other locations considered, also show an increase with time till the maximum values are attained, and, then, indicate a decrease as for the surface temperature. The decrease in temperature is expected from the decrease in heat transfer to the body as the film thickness increases. The energy lost at the bottom becomes larger than that gained at the upper surface. Obviously, $t(x)$ is greater than t_i for $x < b$, even after a very large time. The exactly similar behaviour is observed for any other value of r.

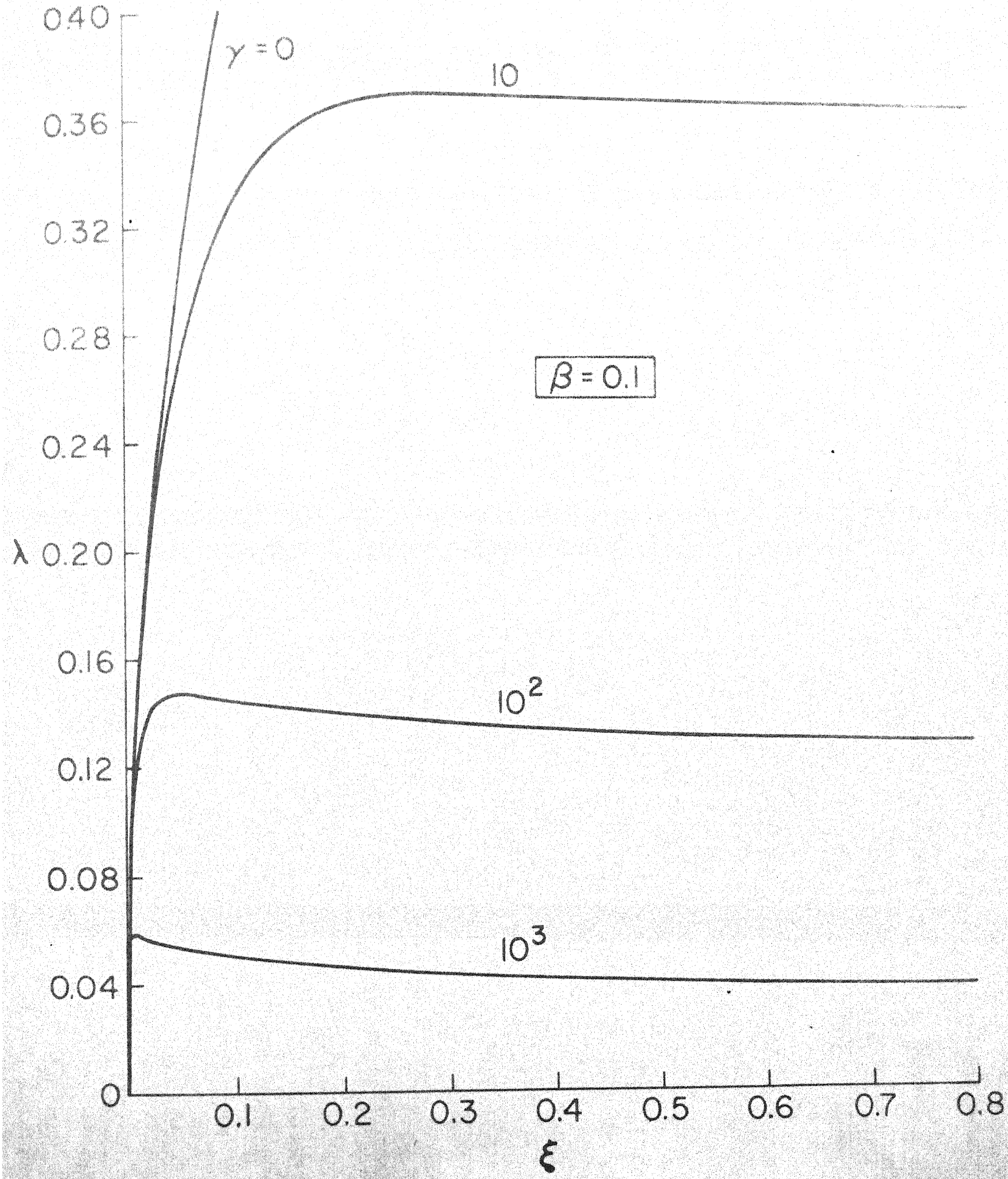

Fig.16 Effect of γ on λ for transient film-condensation on a plate, of finite thickness and bottom surface at a fixed temperature

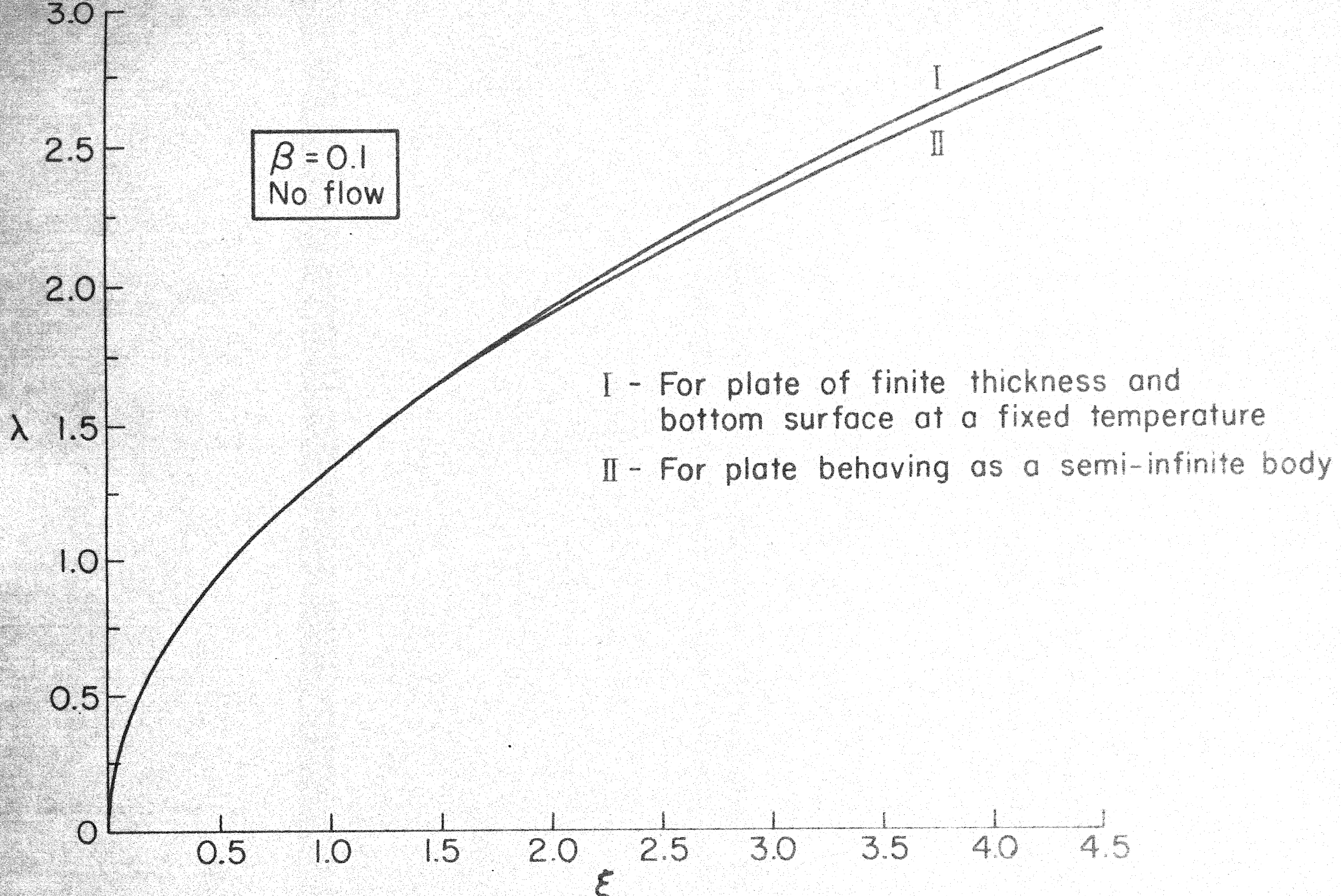

Fig.17 A comparison between the results for transient film-condensation on a plate behaving as a semi-infinite body and a plate of finite thickness and bottom surface at a fixed temperature

For the gravity dominated flow, the film thickness attains a maximum value after a certain time and then decreases to a constant value which is maintained for all times. This steady state indicates that whatever vapour is condensed, flows out and the heat gained at the upper surface is lost at the bottom surface. As γ increases the maximum film thickness decreases, as shown in Fig. 16. Similarly, the temperatures at different locations in the plate, for $\gamma > 0$, are constant after a certain time, as can be seen from Fig. 15. A comparison has also been made between the film thickness solutions for a semi-infinite plate and those for a plate of finite thickness, as presented in Fig. 17. Clearly, the difference is small over the time period considered.

(e) Plate of Finite Thickness and Bottom Surface Insulated:

The solutions for this problem are an improvement over the solutions for the plate with insulated bottom surface and negligible temperature variation across its thickness. The behaviour of the physical variables is the same except for a small difference in the values, resulting from the assumed quadratic temperature profile in the plate. Fig. 18 shows the effect of r on the surface temperature ϕ_o and Figure 19 presents the behaviour of the film thickness for a range of γ. As can be seen, the temperature approaches the saturation vapour temperature and the film thickness gradually goes to zero for the gravity dominated flows.

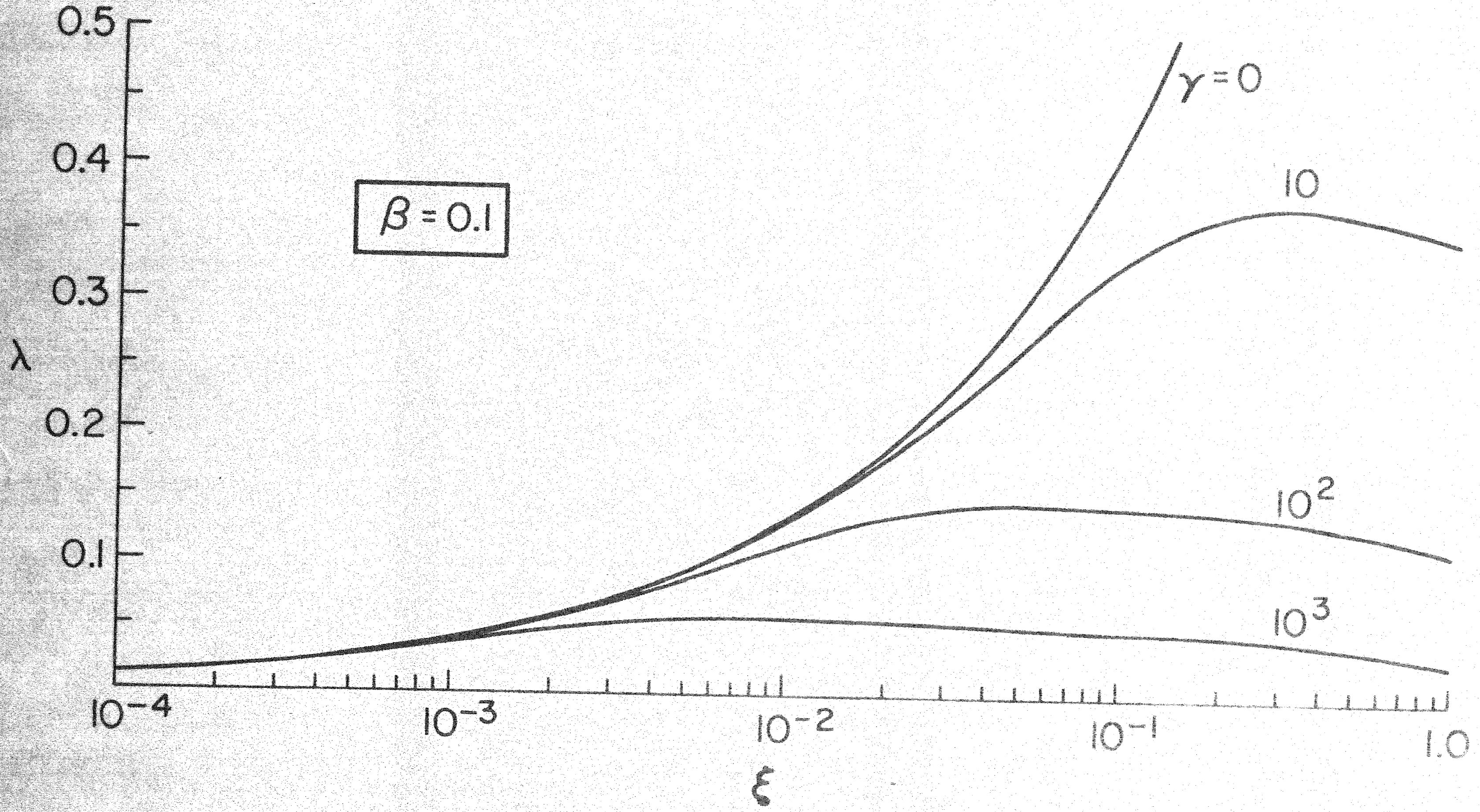

Fig.19 Effect of γ on λ for transient film-condensation on a horizontal plate, of finite thickness and bottom surface insulated

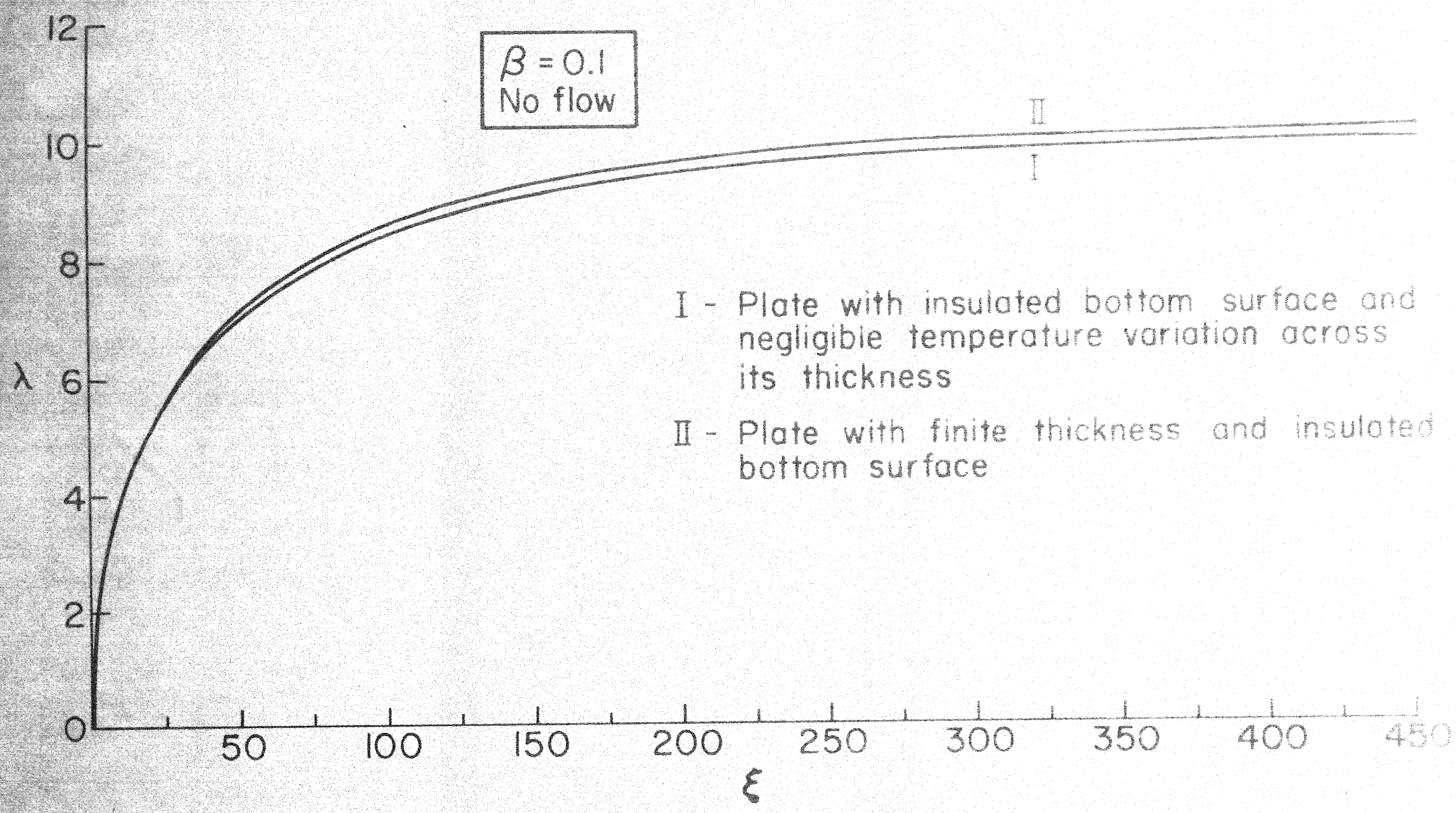

Fig.20 A comparison between results for transient film-condensation a plate, of uniform thickness and insulated bottom surface, wi and without a temperature variation across its thickness

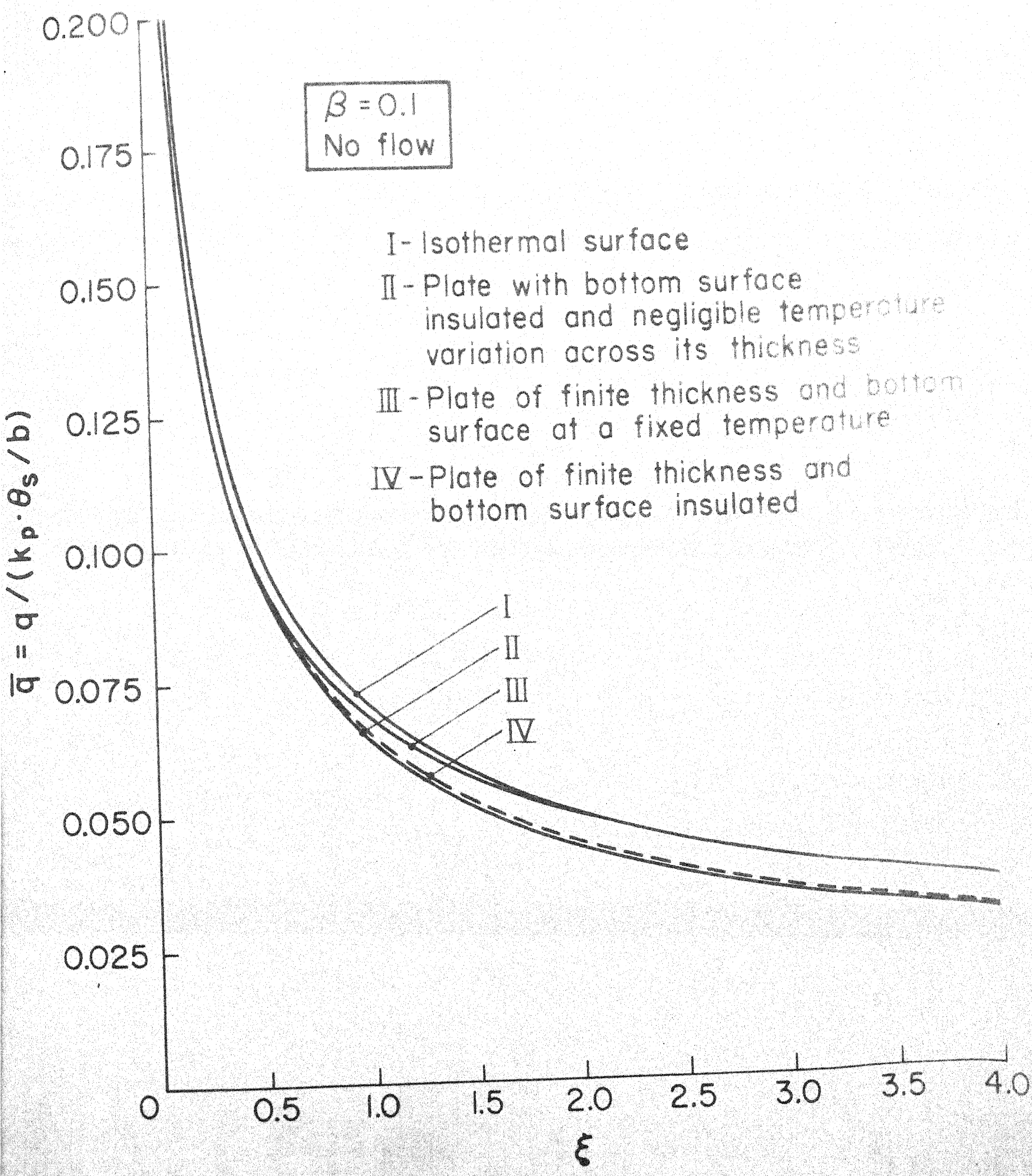

Fig.21 Effect of various boundary conditions on the heat transfer rates for transient film-condensation on a horizontal plate

A comparison has been made between the results obtained for the condensation on a plate, of uniform thickness and insulated bottom surface, with and without a temperature variation across its thickness, as presented in Fig. 20. The behaviour of the film thickness is quite similar in the two cases, though the temperature variation in the plate is obviously quite different.

(f) Heat Flux:

Fig. 21 shows the variation in heat transfer rates, with time, for various boundary conditions of the plate, for the no flow condition. Heat transfer rates are found to decrease sharply with time for all the cases. For the no flow condition, the film thickness increases with time and this would obviously result in the reduction of heat transfer rates. This result would be applicable for short times when flow has not started due to surface tension effects and for circumstances where a wall obstructs the condensate flow. This figure simply shows the similarity in the heat transfer results for the various cases. Similar curves may be obtained for other circumstances.

CHAPTER III

LAMINAR FILM CONDENSATION FLOW ON A HORIZONTAL ISOTHERMAL SURFACE

3.1 Analysis:

The present chapter discusses the analysis and the numerical results for laminar film condensation flow on a horizontal isothermal surface (Fig. 22), subject to the approximations discussed in section (2.1). The analysis ignores any disturbances, caused by the condensation of

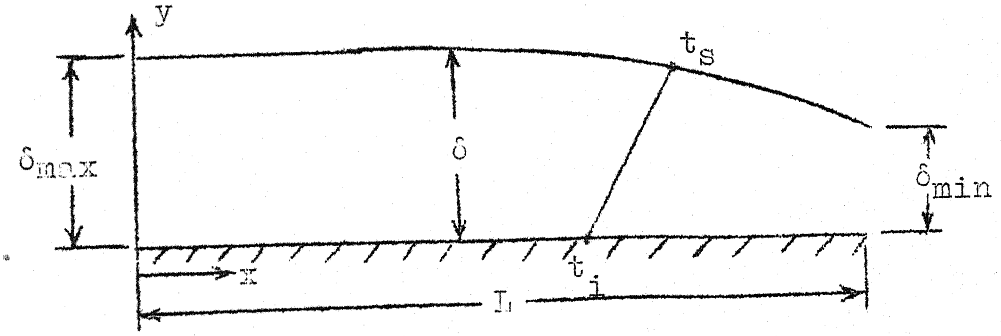

Fig. 22 Physical model and co-ordinate system for laminar film condensation flow.

vapour, on the velocity and temperature of the condensate. The complete analysis has been divided into three parts, as already discussed in Chapter I.

3.1.1 Transient Film Condensation Without Flow:

When a horizontal isothermal surface, at a temperature t_i, is exposed to a saturated vapour, at a uniform temperature t_s, corresponding to its pressure, condensation occurs. The

liquid, thus condensed, does not flow out across the edges of the condensing surface unless the condensate film is sufficiently thick, so that the hydrostatic forces overcome the surface tension and viscous forces. Hence, transient film condensation, with no flow occurs initially and is considered first, as given below.

The thickness of the condensate film, δ, which is zero at time $\tau = 0$, will continue to increase till the flow starts. Therefore, from Eq. (2.3), the condensate film thickness for no flow, is obtained as:

$$\delta = \sqrt{\frac{2k_l \, (t_s - t_i)}{\rho \, h_{fg}} \, \tau} \qquad (3.1)$$

If the condensing surface is terminated by a sharp edge, the film is essentially a large, two-dimensional drop [21], whose height increases with time. The shape of this drop, just before the flow starts, is determined by a balance between the gravity and surface tension forces. Bankoff [25] has analyzed the problem of spreading of large drops and both his analytical results and experimental observations show that the surface tension controlled film is of practically constant thickness over most of its length, when δ_{max}, the film thickness at $x = 0$, is small as compared to the length of the plate. For water with a contact angle of $\pi/2$, for example, the film thickness attains 99 percent of its maximum value within 3 δ_{max} of the edge [21].

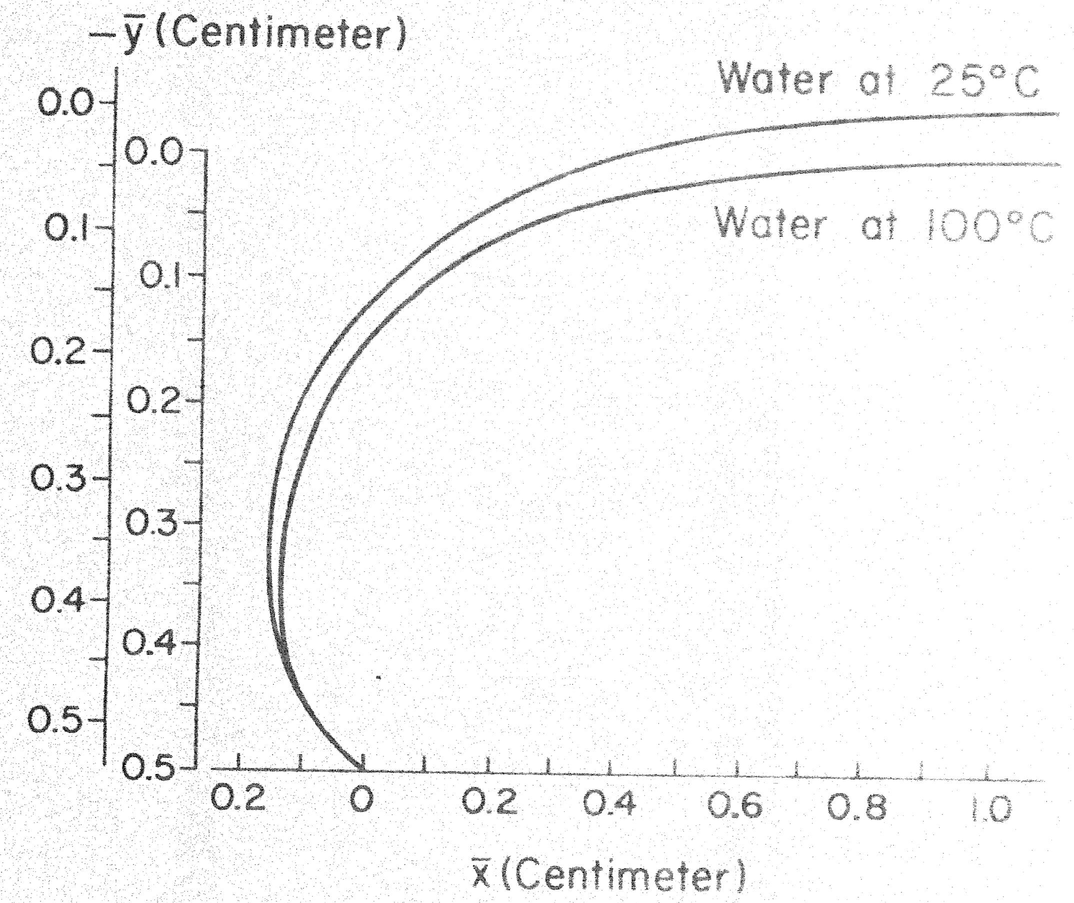

Fig. 23 Profiles of a large drop of water, at various temperatures, advancing to the left [25]

The expression which determines the film profile (Fig. 23) has been obtained by Bankoff [25] and is given by:

$$\bar{x} = B^{\frac{1}{2}} \log \left[- \frac{2B^{\frac{1}{2}}}{\bar{y}} \left(\sqrt{1 - \frac{\bar{y}^2}{4B}} + 1 \right) \right] - 2 B^{\frac{1}{2}} \sqrt{1 - \frac{\bar{y}^2}{4B}} + \text{Constant} \tag{3.2}$$

where,

$$B = \frac{\sigma}{(\rho - \rho_v)g} \tag{3.3}$$

and, σ is the surface tension. The sign in front of the square root terms would be reversed for a drop advancing to the right. For the co-ordinate system with $\bar{x} = 0$, when $\bar{y} = 2 B^{\frac{1}{2}}$, the constant in Eq. (3.2) vanishes [25]. Fig. 23 presents the profile of a large drop of water, at different temperatures, advancing to the left.

If end effects are ignored, Eq. (3.2) will give the film thickness δ_1, at x = 0, which must be attained before the flow occurs. As the variation in the film thickness at the end of the condensing surface is very small, as discussed earlier, we may obtain the time τ_1 required for the film thickness to be δ_1, assuming it to be uniform over the entire surface, from Eq. (3.1).

3.1.2 Transient, Laminar Film Condensation Flow:

After the flow of the condensate has just started, it would undergo a transient process. In this section, the transient, laminar film condensation flow has been considered.

Assuming the flow to be unidirectional, a simple balance between the pressure gradient and the viscous and surface tension forces, at any instant, gives,

$$\mu \frac{\partial^2 u}{\partial y^2} - (\rho - \rho_v)\, g \,\frac{\partial \delta}{\partial x} - \frac{\partial p_\sigma}{\partial x} = 0 \qquad (3.4)$$

where, u is the velocity in the x-direction, p_σ the pressure due to surface tension forces, μ the viscosity of the condensate fluid and δ a function of x and τ.

Since the thickness of the film is assumed to change substantially only in the direction of flow, i.e., in the x direction, p_σ is given by [32, 33]:

$$p_\sigma = \frac{\sigma}{R_x} \qquad (3.5)$$

where, R_x is the radius of curvature of the film surface and is related to its thickness by the well-known equation [32]:

$$R_x = -\left[1 + \left(\frac{\partial \delta}{\partial x}\right)^2\right]^{3/2} \Big/ \frac{\partial^2 \delta}{\partial x^2} \qquad (3.6)$$

As $\frac{\partial \delta}{\partial x} \ll 1$, we may assume that

$$p_\sigma = -\sigma \frac{\partial^2 \delta}{\partial x^2} \qquad (3.7)$$

Hence Eq. (3.4) may be written as:

$$\mu \frac{\partial^2 u}{\partial y^2} - (\rho - \rho_v)\, g \frac{\partial \delta}{\partial x} + \sigma \frac{\partial^3 \delta}{\partial x^3} = 0 \qquad (3.8)$$

As the velocity at y = 0 is zero (Fig. 22), and the interfacial shear between the liquid film and the vapour has been assumed to be negligible, we get,

$$u = 0 \quad \text{at } y = 0 \tag{3.9a}$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = \delta \tag{3.9b}$$

Integrating Eq. (3.8), over y, and using the above boundary conditions, we obtain,

$$u = \frac{1}{\mu} \left(\bar{\rho}\, g \frac{\partial \delta}{\partial x} - \sigma \frac{\partial^3 \delta}{\partial x^3} \right) \left(\frac{y^2}{2} - y\delta \right) \tag{3.10}$$

where,

$$\bar{\rho} = (\rho - \rho_v) \tag{3.11}$$

For an assumed linear temperature profile in the condensate film, we write,

$$\frac{\partial t}{\partial y} = \frac{t_s - t_i}{\delta(x,\tau)} = \frac{\Delta t}{\delta(x, \tau)} \tag{3.12}$$

where,

$$\Delta t = t_s - t_i$$

Integrating Eq. (3.12) from y to δ, we get,

$$t_s - t = \left(1 - \frac{y}{\delta}\right) \Delta t \tag{3.13}$$

Considering the energy balance of an element of the film of height δ and thickness dx, depth being unity, we write,

$$\frac{\partial m}{\partial x} \cdot h_{fg} - k_1 \frac{\partial t}{\partial y}\bigg|_{y=0} = \rho\, C_{p_1} \left[\frac{\partial}{\partial \tau} \int_0^\delta (t - t_s)\, dy + \frac{\partial}{\partial x} \int_0^\delta u\, (t - t_s)\, dy \right] \tag{3.14}$$

where,

$$\frac{\partial m}{\partial x} = \text{Condensation rate per unit length.}$$

Using Eqs. (3.10), (3.12) and (3.13), the above equation may be written as:

$$\frac{\partial m}{\partial x} = \frac{\Delta t}{h_{fg}} \left[\frac{k_1}{\delta} - \rho C_{p_1} \frac{\partial \delta}{\partial \tau} + \frac{\rho C_{p_1}}{8\mu} \frac{\partial}{\partial x} \{\delta^3 \ (\bar{\rho} g \frac{\partial \delta}{\partial x} - \sigma \frac{\partial^3 \delta}{\partial x^3})\} \right] \tag{3.15}$$

Now, considering the mass conservation for the element, we obtain,

$$\frac{\partial m}{\partial x} = \frac{\partial}{\partial x} \left[\int_0^\delta \rho u \, dy + \rho \frac{\partial \delta}{\partial \tau} \right] \tag{3.16}$$

Introduction of Eq. (3.10) into the above equation yields,

$$\frac{\partial m}{\partial x} = \rho \frac{\partial \delta}{\partial \tau} - \frac{1}{3} \frac{\rho}{\mu} \frac{\partial}{\partial x} \left[\delta^3 \ (\bar{\rho} g \frac{\partial \delta}{\partial x} - \sigma \frac{\partial^3 \delta}{\partial x^3}) \right] \tag{3.17}$$

From Eqs. (3.15) and (3.17), we get,

$$\frac{\rho (h_{fg} + \frac{1}{2} C_{p_1} \Delta t)}{k_1 \Delta t} \delta \frac{\partial \delta}{\partial \tau} - \frac{\rho}{3\mu} \frac{(h_{fg} + \frac{3}{8} C_{p_1} \Delta t)}{k_1 \Delta t} \delta \left[\frac{\partial}{\partial x} \{ \delta^3 \ (\bar{\rho} g \frac{\partial \delta}{\partial x} - \sigma \frac{\partial^3 \delta}{\partial x^3})\}\right] = 1 \tag{3.18}$$

or,

$$\frac{3 \rho \nu (1 + \frac{1}{8} \frac{C_{p_1} \Delta t}{h'_{fg}})}{\bar{\rho} g} \delta \frac{\partial \delta}{\partial \tau} + \delta^3 \left[\frac{\sigma}{\bar{\rho} g} \{ \delta \frac{\partial^4 \delta}{\partial x^4} + 3 \frac{\partial \delta}{\partial x} \frac{\partial^3 \delta}{\partial x^3} \} - \{ \delta \frac{\partial^2 \delta}{\partial x^2} + 3 (\frac{\partial \delta}{\partial x})^2 \}\right] = \frac{3 \nu k_1 \Delta t}{\bar{\rho} g h'_{fg}} \tag{3.19}$$

which, in terms of dimensionless distance, X, yields,

$$\frac{3\rho\,\nu\,(1+\frac{1}{8}\,\frac{C_{p_1}\,\Delta t}{h'_{fg}})\,L^2}{\bar{\rho}\,g}\,\delta\,\frac{\partial\delta}{\partial\tau}+\delta^3\,[\,\frac{\sigma}{\bar{\rho}\,gL^2}\{\,\delta\,\frac{\partial^4\delta}{\partial X^4}$$

$$+\,3\,\frac{\partial\delta}{\partial X}\,\frac{\partial^3\delta}{\partial X^3}\,\}-\{\,\delta\,\frac{\partial^2\delta}{\partial X^2}+3\,(\frac{\partial\delta}{\partial X})^2\,\,]=\frac{3\,\nu\,k_1\,\Delta t}{\bar{\rho}\,g\,h'_{fg}}\,L^2 \tag{3.20}$$

where,

$$X = x/L \tag{3.21}$$

Hence, we obtain,

$$R\,\delta\dot{\delta}+\delta^3\,[\frac{1}{Bo}\,(\delta\delta^{IV}+3\delta'\,\delta''')-(\delta\delta''+3\delta'^2)]=KL^2 \tag{3.22}$$

where,

$$R=\frac{3\,\rho\,\nu\,(1+\frac{1}{8}\,\frac{C_{p_1}\,\Delta t}{h'_{fg}})\,L^2}{\bar{\rho}\,g} \tag{3.23}$$

$$K=\frac{3\,\nu\,k_1\,\Delta t}{\bar{\rho}\,g\,h'_{fg}} \tag{3.24}$$

Bond Number, $Bo = \dfrac{\bar{\rho}gL^2}{\sigma}$ (3.25)

and $\dot{\delta} = \dfrac{\partial\delta}{\partial\tau}$ (3.26a)

$$\delta^{IV} = \frac{\partial^4\delta}{\partial X^4} \tag{3.26b}$$

If surface tension effects are ignored, or for large values of Bond number, Eq. (3.22) yields,

$$R\,\delta\,\dot{\delta} - \delta^3\,(\delta\delta'' + 3\,\delta'^2) = KL^2 \qquad (3.27)$$

Now, proper conditions must be determined to solve the above differential equations. For a condensing surface terminated by sharp edges, the initial condition is obtained from the section (3.1.1), i.e.,

$$\delta\,(X,\,0) = \delta_1 \qquad (3.28)$$

For other end conditions, i.e., for a rounded fall at the end of the condensing surface, we must obtain the film profile, necessary for the flow to start, by some other methods. For certain fall conditions, it may be expected that the film thickness at the time of flow to start is quite small.

Now, since the flow is symmetric on both sides of the condensing surface, we get,

$$\frac{\partial\,\delta}{\partial\,X}\,(0,\,\tau) = 0 \qquad (3.29)$$

The other boundary condition, required for the solution of Eq. (3.26), must be known at the end of the condensing surface. For certain fall conditions, we may take η_{min} as constant, where,

$$\eta = \frac{\delta}{\delta_{max}} = \frac{\delta(X,\,\tau)}{\delta(0,\,\tau)} \qquad (3.30)$$

and, hence,

$$\eta_{min} = \frac{\delta_{min}}{\delta_{max}} = \frac{\delta(1, \tau)}{\delta(0, \tau)} \tag{3.31}$$

The other possibility is to take

$$\delta' (1, \tau) = \text{Constant} \tag{3.32}$$

as a boundary condition, because the rounded fall at the end has a definite slope. Using the initial condition, Eq.(3.29) and either of Eqs. (3.31) and (3.32), as applicable, we may obtain the transient solution from Eq. (3.27).

If surface tension effects are also considered, we must solve the equation (3.22), which requires two more boundary conditions to be known. As the flow is symmetric on both sides of the condensing surface, the fluid at $x = 0$ will be at zero velocity, and, hence, from Eq. (3.10), we get,

$$\frac{\partial^3 \delta}{\partial x^3} (0, \tau) = 0 \tag{3.33}$$

Therefore, for a fall condition, where, both η_{min} and $\delta'(1,\tau)$ are constant, Eq.(3.22) may be solved using the initial condition and Eqs. (3.30) through (3.33).

Since, the temperature profile in the condensate film is assumed to be linear, the local heat transfer co-efficient h_x is given by:

$$h_x = h(x, \tau) = \frac{k_1}{\delta(x, \tau)} \tag{3.34}$$

And, hence, the local Nusselt number is,

$$Nu_X = \frac{X}{\delta(X, \tau)} \tag{3.35}$$

3.1.3 Steady, Laminar Film Condensation Flow:

For negligible surface tension effects, or for large values of Bo, Eq. (3.27) yields the condensate film profile for steady, laminar film condensation flow as:

$$\delta^3 [\delta\delta'' + 3(\delta')^2] = -KL^2 \tag{3.36}$$

with the conditions:

$$\delta'(0) = 0 \tag{3.37}$$

$$\eta_{min} = \text{Constant} \tag{3.38}$$

If the transient solution is obtained, δ_{max} will be a known quantity, and as such,

$$\delta(X, 0) = \delta_{max} \tag{3.39}$$

may be used as a boundary condition, instead of Eq.(3.38). Even, if the film thickness at any X is known, Eq.(3.36) may be solved with the help of Eq. (3.37).

Nimmo and Leppert [20,21] have also analyzed the steady, laminar film condensation flow, with negligible surface tension effect, and, have obtained an equation similar to Eq. (3.36). Using the condition that the film thickness at $X = 0$, δ_{max}, is known and that given by Eq. (3.37), they have

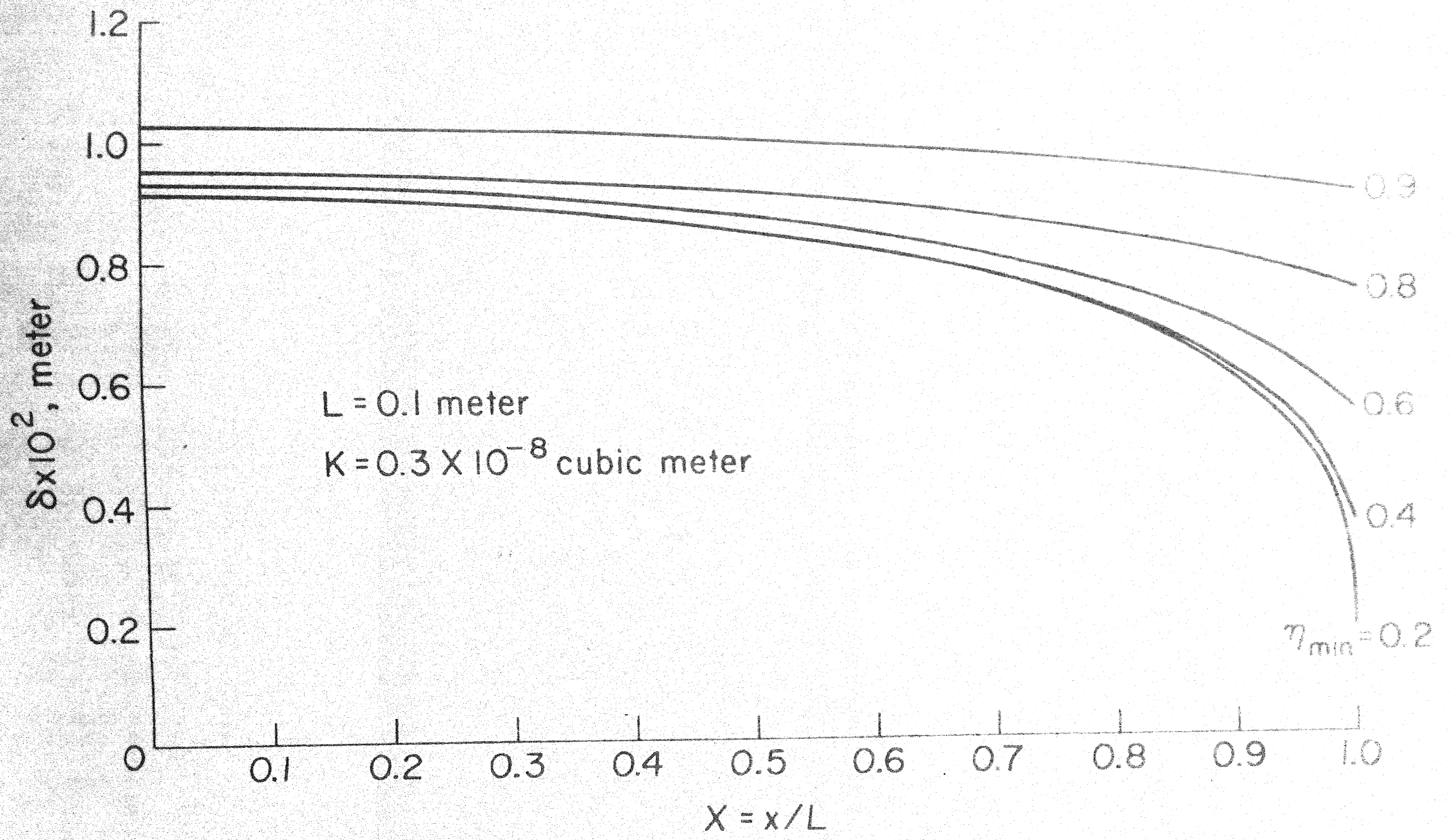

Fig. 24 Variation in condensate film profile for various fall conditions η_{min} for steady, laminar film condensation flow on a horizontal isothermal surface

presented the exact solution of Eq. (3.36) in terms of elliptic integral.

If surface tension effects are considered, Eq.(3.22) yields,

$$\delta^3 \left[\frac{1}{Bo} (\delta\delta^{iv} + 3\,\delta'\delta''') - (\delta\delta'' + 3\delta'^2)\right] = KL^2 \qquad (3.40)$$

The above equation may be solved employing the conditions,

$$\delta'''(0) = 0 \qquad (3.41)$$

and $\delta'\ (1) =$ Constant (3.42)

in combination with the other two boundary conditions discussed above. It may be noted that Eq. (3.36) or Eq.(3.40) may be directly derived from the equations of motion, mass conservation and energy balance. Again, in the present case, the local heat transfer co-efficient is

$$h_x = \frac{k_l}{\delta(x)} \qquad (3.43)$$

and, the local Nusselt number is

$$Nu_x = \frac{x}{\delta(x)} \qquad (3.44)$$

3.2 Results and Discussion:

(a) Steady, Laminar Film Condensation Flow:

The condensate film profile for steady, laminar film condensation flow, with negligible surface tension effects, is given by Eq. (3.36). Fig. 24 presents the numerically obtained film profile for various values of η_{min}. There is

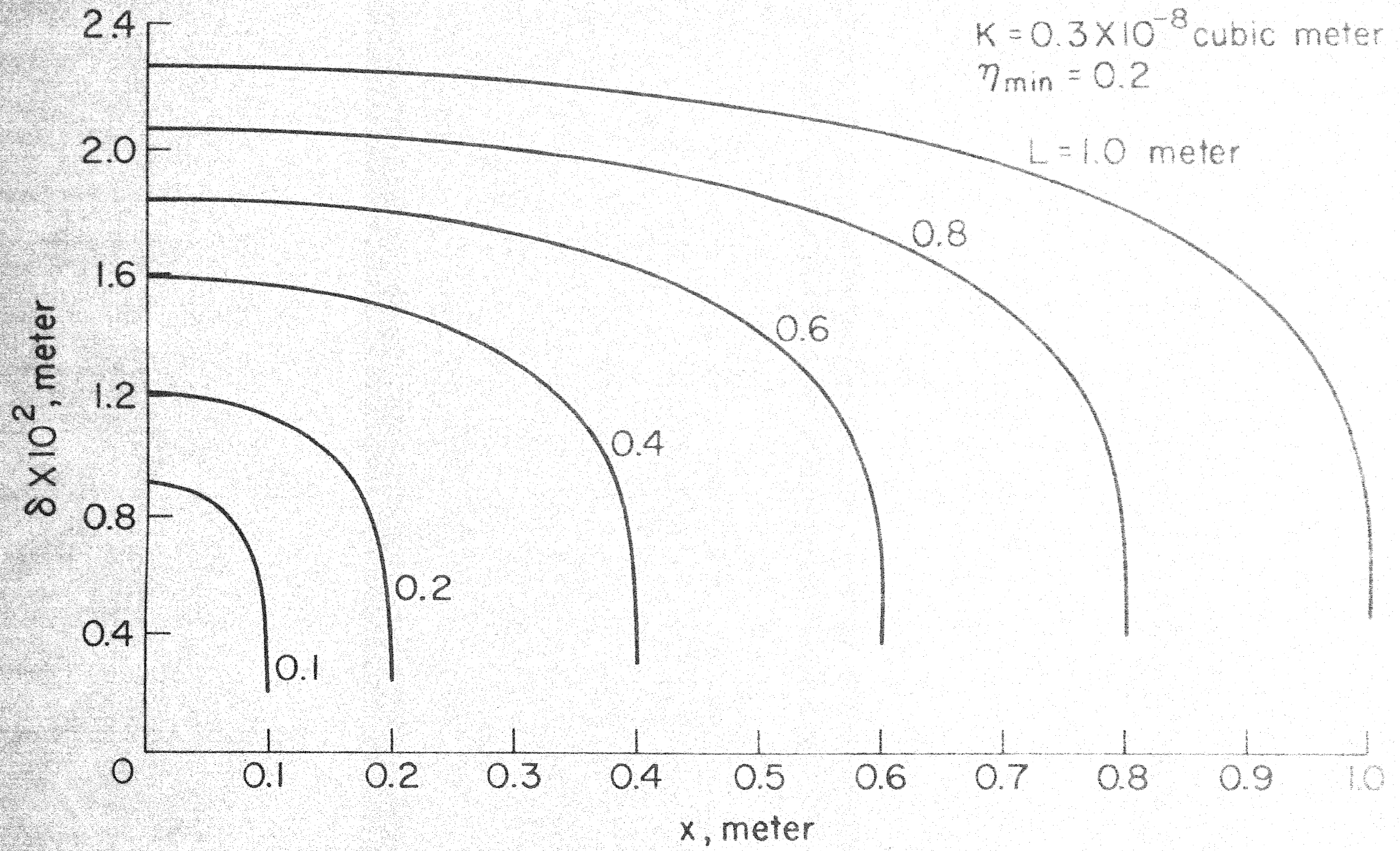

Fig.25 Effect of length of the condensing surface L on condensate film profile for steady, laminar film condensation flow on horizontal isothermal surface

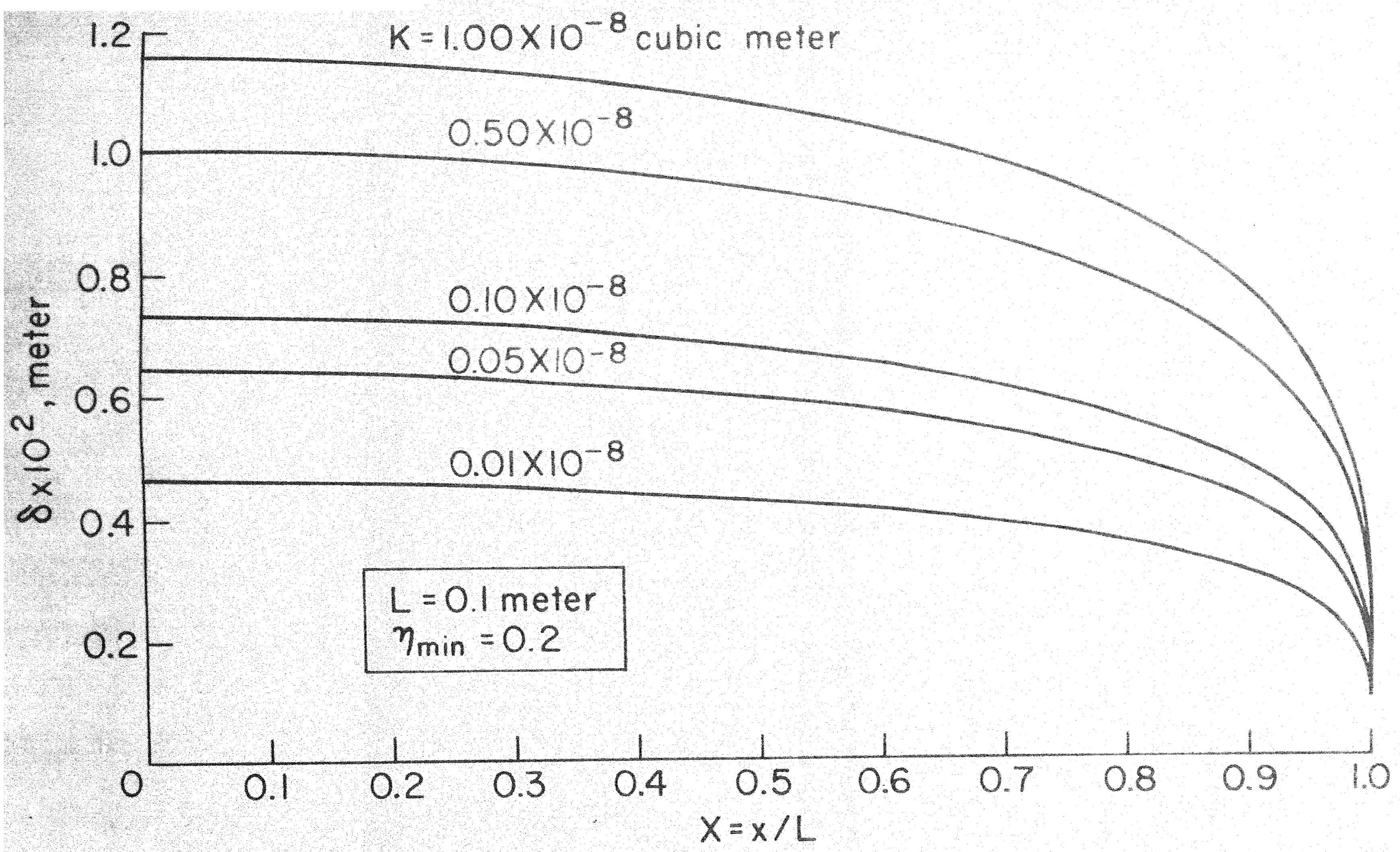

Fig. 26 Effect of variation of K on condensate film profile for steady, laminar film condensation flow on a horizontal isothermal surface

very little difference in the film profiles for $\eta_{min} \leq 0.4$. The same behaviour has also been reported by Nimmo and Leppert [21]. In this range of η_{min}, δ_{max} is almost a constant. Fig. 24 also shows the variation in the maximum film thickness at $X = 0$, δ_{max}, with respect to η_{min}. As η_{min} increases, δ_{max}, and, hence δ, rises, reducing the condensation rate and, therefore, decreasing the flow rate. Alternatively, the film profile and, hence η_{min}, would be known if δ_{max} is known.

As is evident from Eq. (3.36), the film profile is a strong function of the length of the condensing surface, L. Fig. 25 shows the condensate film profiles for a range of L. It is observed that δ_{max} increases as L is increased, which is expected, since the driving force must be greater for greater L. Also a larger length of the condensing surface L gives a greater condensation rate and, hence, a larger flow rate.

The film profile and the rate of condensation are also affected by the properties of the condensing vapour and by the difference between vapour saturation temperature and surface temperature, Δt. Fig. 26 presents the condensate film profiles for various values of K, defined in Eq.(3.24). The value of K increases with the viscous forces, the thermal conductivity and the temperature difference, Δt, and decreases when gravity forces or latent heat of the condensing

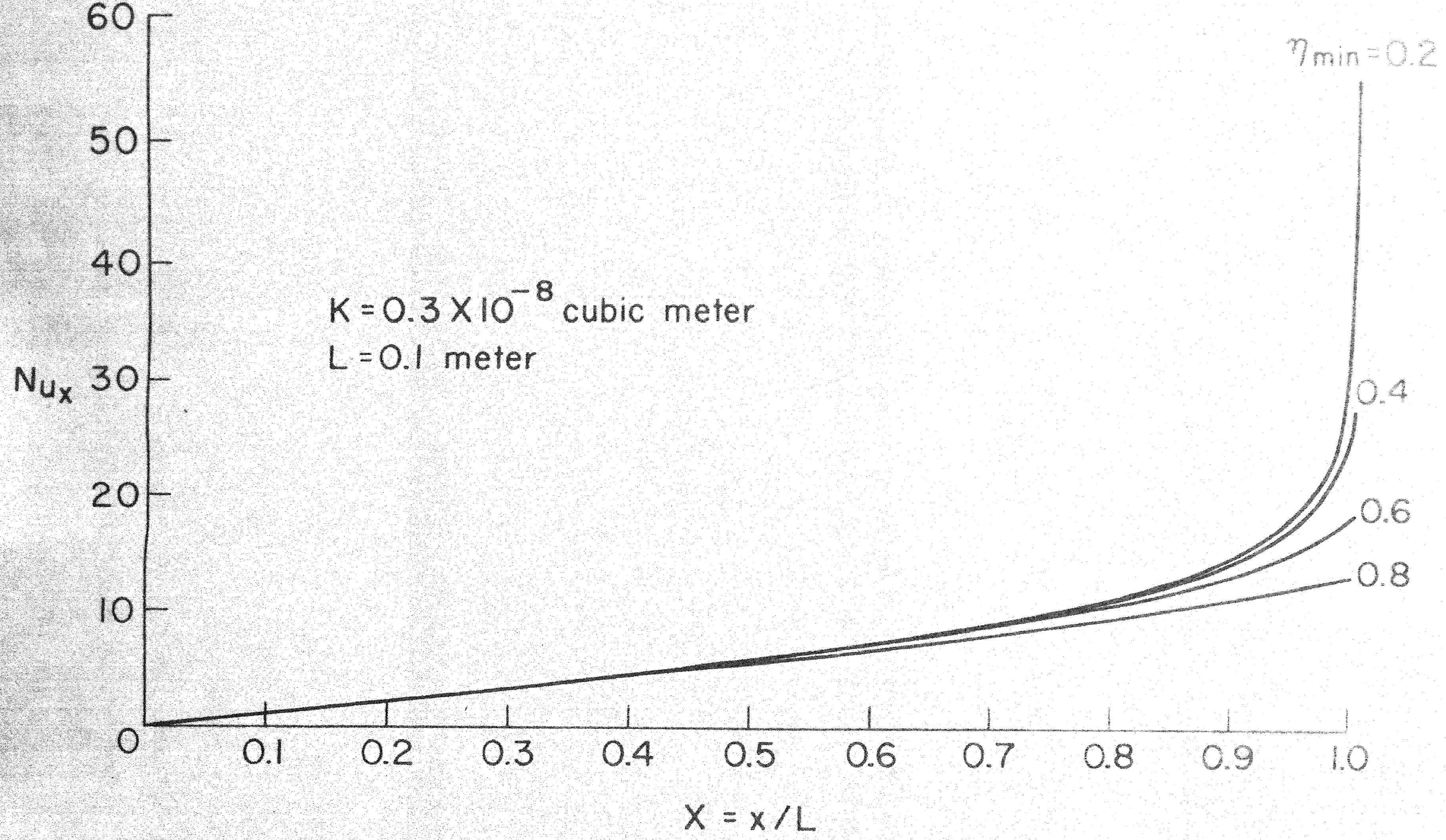

Fig.27 Variation of local Nusselt number Nu_x, for a range of η_{min}, for steady, laminar film condensation flow on a horizontal isothermal surface

vapour increases. And, hence, the value of K is large for a highly viscous condensing fluid. Fig. 26 shows that the film thickness increases with K and, thus, the condensation rate is reduced.

The local Nusselt number, Nu_x, at various locations has been presented in Fig. 27, for a range of η_{min} keeping K and L as constant. As η_{min} decreases, Nu_x at the end of the condensing surface increases, while the variation in its value at low values of X is quite small. The value of mean Nusselt number, Nu_m, for steam when $\eta_{min} = 0.4$, is found to be within 2% of 0.82 $Sh^{1/5}$ as reported by Nimmo and Leppert [21] but the value is appreciably larger for $\eta_{min} \leq 0.2$.

The effect of surface tension terms in Eq.(3.40), for steady, laminar flow, depends on the values of Bond number, Bo. The effects are expected to be larger for smaller values of Bo. Eq. (3.40) has been solved by finite - difference techniques for various values of η_{min} and the length of the condensing surface, L. The results show that the effect of surface tension is negligible, even for very small values of the Bond number. The change in maximum film thickness, δ_{max}, is within 0.1 percent for Bo ≥ 0.0001, when η_{min} equals 0.6 and L is 4 cms. The effect of surface tension diminishes, largely due to assumed round fall at the end of the condensing surface. For the condensing surface

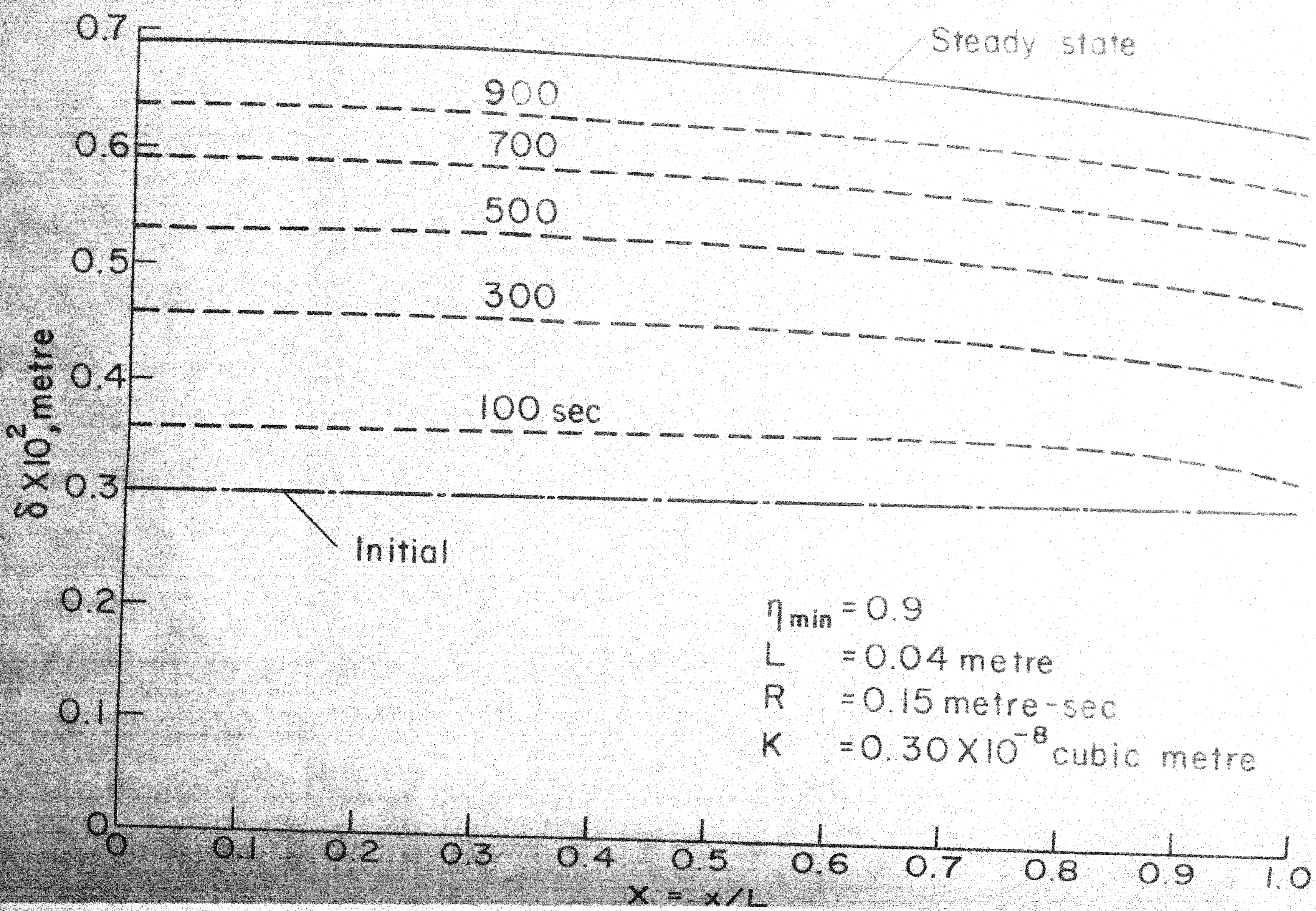

Fig. 28 Condensate film profiles for transient, laminar film condensation flow on a horizontal isothermal surface

terminated by a sharp edge, the effect is expected to be considerable, but due to the lack of knowledge of proper boundary conditions, it could not be considered.

(b) Transient, Laminar Film Condensation Flow:

As is clear from the Eqs. (3.22) and (3.27), the transient effects on condensate film profile are controlled by a parameter R, given by Eq. (3.27). The parameter R is the same as that obtained by Sparrow and Siegel [18] for their analysis of transient laminar film condensation on a vertical surface. The values of R depend on the condensing vapour properties, length of the condensing surface, L, and acceleration due to gravity, g and, as such, the transient effects will be larger for highly viscous condensing fluids, or for a field of low gravity. Fig. 28 presents the condensate film profile, with negligible surface tension, for various times, obtained for the initial condition that the film thickness is uniform at $\tau = 0$ and, after the flow starts, η_{min} is a constant. It is observed that the rate of increase in film thickness decreases with time, and, hence, the local heat transfer coefficient h_x, also decreases. The effect of surface tension is expected to be the same, as discussed for the steady, laminar flow.

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CHAPTER IV

CONCLUSIONS

The results of the present investigation are important as much of this work is the first known treatment of many of the problems. For the transient film condensation on the upper surface of a horizontal plate, which is considered for five different boundary conditions, it is emphasized that there are many possible physical mechanisms which could govern run-off. The analysis is done for two inviscid flow conditions, one, a specified fraction of the rate of condensation, occurring at any time, flowing out across the edges of the condensing surface at that instant, and, second, the out-flow dominated by gravity. The theoretical results are shown to depend on two dimensionless parameters, one for condensation, β, except for the case of isothermal surface, and the other for the flow, r or γ. The situation of no-flow arises when the flow parameter is zero, and, hence, transient film condensation with no run-off depends on only one dimensionless parameter, β. This result would be applicable for short times when the flow has not started due to surface tension effects and for circumstances where a wall obstructs the condensate flow.

Laminar film condensation flow on a horizontal isothermal surface is analyzed in three stages: (a) transient film condensation without flow, (b) transient, laminar flow and (c) steady, laminar flow. The analyses for transient and steady laminar flows consider the surface tension effects also. Fourth order non-linear differential equations are obtained for the film thickness in both the transient and the laminar cases. Negligible surface tension, or large values of Bond number reduce them to second order systems. The steady, laminar condensation flow is shown to depend upon a parameter, K, which is a function of vapour properties and the difference between the temperatures of the vapour and the condensing surface. The condensate film profile for the above flow is found to be a strong function of the edge condition and the film thickness increases with length of the condensing surface, and the parameter K. The transient response of the film thickness is controlled by another parameter R, which, also, depends on the vapour properties, length of the condensing surface and the acceleration due to gravity. The analysis and results show that the transient effects are large for highly viscous condensing fluids and for low gravity fields. It is also observed that for the condensing surface terminated by a round fall, the surface tension effects are negligible even for very small values of Bond number.

Recommendations for Future Work:

For the transient film condensation, combined with conduction, on a horizontal body, the future work may be concentrated in the following areas: Theory may be developed for the flow fraction parameter r as a function of time, and experiments need to be performed on substantiating the theory developed.

For the laminar film condensation flow on a horizontal isothermal surface, further work is needed on several aspects.

For transient film condensation, without flow, sufficient information must be obtained for the conditions just before the start of the flow, for various edge conditions.

By experimentation or analysis, information should be obtained for specifying the boundary condition at the edge of the condensing surface, terminated by a sharp edge, in the absence of which transient and steady state solutions could not be obtained for the sharp edge case. The effects of R and K need to be further examined for transient flow. The effect of surfaces tension, particularly for a sharp edged surface, needs to be studied further over a range of Bond number.

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